# WEAK DECAYS OF HEAVY QUARKS

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We review some aspects of weak decays of hadrons containing one heavy quark. The main emphasis is on $B$ physics, in particular in the framework of the Heavy Quark Effective Theory.

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# 1 Motivations

In the last decades we have witnessed an impressive progress in our understanding of elementary particle physics phenomena. On the one hand, more and more powerful experimental tools became available, on the other a deeper insight in the theoretical understanding of elementary dynamics has been gained. We have reached a point where the Standard Model describing strong and electroweak interactions of elementary particles has obtained unforeseen confirmations and, simultaneously, the many questions which still remain open push us to look beyond it. This is the frontier of particle physics, to which all of this book is devoted.

Among the many unanswered questions, the mechanism of electroweak symmetry breaking plays a primary role – the Higgs particle which enters in the Standard Model in the description of such a mechanism has not yet been detected. In the fermion sector, it is mostly unclear as to the observed hierarchy between the masses of the particles, in particular it is being debated whether or not we can still believe that neutrinos are massless; as far as quarks are concerned, problems already arise in trying to define their masses, due to the confining strength of QCD interactions in the long distance regime.

Another central question is how do quarks participate in weak decays: i.e. the actual understanding of the mixing pattern. This last point has especially attracted attention both from experimentalists and theorists. The Standard Model describes such mixing by introducing a unitary matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The complex nature of this matrix is the only source of CP violation within the Standard Model: many efforts are devoted to understanding whether this is sufficient to describe all CP violating phenomena observed till now and all those which will hopefully be observed in the nearby future. This is considered one of the most fertile grounds to reveal physics beyond the Standard Model. The requirement of CP violating processes as one of the necessary conditions to generate the observed baryon-antibaryon asymmetry [1] makes this topic extremely appealing also from the standpoint of cosmology.

Weak decays are concerned with all these and many other left aside questions. The phenomenology of such decays is impressively rich, and a huge number of data has been collected at the numerous experimental facilities operating in the past and at present all over the world. On the other hand, most of these data come from hadronic processes, and hence quantum chromodynamics (QCD) enters into the game. QCD is for sure the less easily tractable theory in elementary particle physics. While the perturbation theory has provided us with a great predictive power in quantum electrodynamics (QED),

the limited realm of applicability of such an approach to the strong interactions precludes to us the possibility to give equally precise predictions in QCD. When facing with weak decays of hadrons, we have to take into account that these objects are bound states of quarks and gluons, which is a highly nonperturbative effect. Therefore, the interplay of both interactions should always be considered, and this makes our task more difficult.

From this point of view a particularly favorable sector is the one where hadrons contain a single heavy quark. The large scale is set by the heavy quark mass $m_Q$ which gives us the possibility to exploit asymptotic freedom. Actually, useful considerations can be derived when considering the limit $m_Q \to \infty$. This is realized within the Heavy Quark Effective Theory (HQET) [2–4] which implements systematically new and old ideas on the dynamics of heavy quark systems.[5,6]

The decays of heavy flavored hadrons, in particular $B$ decays, will presumably also show a higher degree of CP violation than the kaon system, where this phenomenon was first observed. This is why new high luminosity machines ($B$ factories) have been built to study CP violation in $B$ decays. The possibility to observe a contribution of non-Standard Model particles, whatever they are, in loop-induced rare $B$ decays, makes this sector appealing from both theoretical and experimental sides.

These are the main motivations to study weak decays of heavy quarks, which we shall review in the following. The first step will be the construction of the effective weak Hamiltonian for $B$ decays. This is useful in many respects: it allows to discuss non leptonic and rare $B$ decays; it will be a guide in the discussion of the Heavy Quark Effective Theory, the construction of which, however, follows a slightly different pattern. Next, we shall briefly recall the formulation of the Heavy Quark Effective Theory, focusing on the aspects which will be relevant for the subsequent analyses. A detailed construction of the Heavy Quark Theory is given elsewhere in this book.[7] As an application of the HQET formalism, we will discuss the relations for leptonic constants and semileptonic decay form factors. Very often in such cases, the large $m_Q$ technique has been used in conjunction with QCD sum rules,[8] which are reviewed elsewhere.[9]

The $1/m_Q$ expansion has also proven to be a very useful tool in the computation of inclusive decays of heavy flavored hadrons. These have attracted considerable interest due to the possibility of reducing the hadronic uncertainty in the sum of exclusive modes and also because of open questions concerning several semileptonic $B$ decay modes and the ratios of the beauty hadron lifetimes. We shall survey these problems in the considered context.

Finally, we shall give some conclusions, with a short comment on some

unaddressed topics.

## 2 Effective Weak Hamiltonian for $B$ decays

In the Standard Model, weak decays of quarks and leptons are mediated by the vector bosons $W^\pm$, $Z^0$. The fact that these particles are very heavy with respect to the energies usually involved in weak decays of hadrons, makes very convenient the use of an effective theory in which these particles are explicitly integrated out of the generating functional of the theory and therefore no longer play a dynamical role. In particular, integrating out the $W$ boson leads to the familiar pointlike four-fermion interaction originally introduced by Fermi in order to describe the nuclear $\beta$ decay. The path which leads from the Standard Model Lagrangian to the Fermi one is nothing but an example of the application of the operator product expansion [10] (OPE). The virtue of using such a description traces back to the interplay of strong and weak interactions. The OPE allows us to write the amplitude relative to a certain weak decay process as the matrix element of an effective Hamiltonian, which is built as a sum of local operators weighted by suitable coefficients (Wilson coefficients). In this context, the operators can be viewed as effective vertices, and the coefficients as effective coupling constants. Since this construction follows the procedure of integrating out the $W$ boson, this particle does not appear in the operators, but its effects are contained in the coefficients. The expansion can roughly be viewed as an expansion both in the inverse powers of $M_W$ and in the dimension of the operators, the most important contribution coming from dimension six four-fermion operators, since higher contributions are suppressed by powers of $p^2/M_W^2$, $p$ being a typical momentum scale of the considered process. In the case of the Fermi theory, it is easy to see what has been just stated: the local four-fermion operator appearing in the Lagrangian is completely independent on $M_W$; however, the Fermi constant contains the dependence on it,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2},$$

$g$ being the $SU(2)_L$ gauge constant.

Most importantly, the OPE succeeds in separating the long distance regime from the short distance one. In fact, both the coefficients and the operators in the OPE depend on a scale $\mu$. This is defined by the fact that we integrate out all the particles with masses above this scale, therefore the operators describe the physics below $\mu$, while the coefficients take into account the physics above it, i.e. the short distance regime. This is why they are usually referred to as "short distance coefficients." However, the choice of $\mu$ is arbitrary, so

that the dependence on it should cancel between operators and coefficients. In this way we have disentangled the nonperturbative ingredients of our process, i.e. the matrix elements of the operators in the OPE, from the purely perturbative ones, the Wilson coefficients. Therefore, a good choice of $\mu$ is obtained requiring that the strong coupling constant is low enough to make meaningful the perturbative calculation. In $B$ and $D$ decays a common choice is usually $\mu = \mathcal{O}(m_b)$, $\mu = \mathcal{O}(m_c)$, respectively. The Wilson coefficients are determined by requiring that, at a definite scale, the amplitudes relevant to a process should coincide in the full and in the effective theory, a procedure called *matching*. The perturbative calculation typically shows the appearance of large logarithms of the kind,

$$\alpha_s(\mu) \ln \left( \frac{M_W^2}{\mu^2} \right),$$

so that, even when $\alpha_s(\mu)$ is a good expansion parameter, this product is $\mathcal{O}(1)$, and therefore the validity of the perturbative expansion is spoiled. The powerful tools of the renormalization group help us to sum up these large logarithms, using what is called the *renormalization group improved perturbation theory*. The leading term in this case stems from the resummation of the terms

$$\left[ \alpha_s(\mu) \ln \left( \frac{M_W^2}{\mu^2} \right) \right]^n$$

(the so-called leading log approximation). In general, at order $m$ in this new expansion, one resums the terms

$$\alpha_s(\mu)^m \left[ \alpha_s(\mu) \ln \left( \frac{M_W^2}{\mu^2} \right) \right]^n.$$

Generally speaking, it is often insufficient to stop at the leading log approximation, and the next-to-leading order corrections should be included, which are actually the truly $\mathcal{O}(\alpha_s)$ corrections with respect to the leading term and which show many interesting features hidden in the leading log approximation.[11]

Keeping in mind the general steps necessary to build an effective Hamiltonian, let us write explicitly the effective Hamiltonian describing nonleptonic weak decays of $B$ mesons:

$$H_{eff}^B = \frac{G_F}{\sqrt{2}} \sum_{U=u,c} V_{Ub} \; [C_1(\mu) \; Q_1^{Ub} + C_2(\mu) \; Q_2^{Ub}] + \text{h.c.} \tag{1}$$

where we have omitted penguin operators and

$$
\begin{aligned}
Q_1^{Ub} &= (\bar{U}b)_{V-A} \left[ (\bar{d}'u)_{V-A} + (\bar{s}'c)_{V-A} \right], \\
Q_2^{Ub} &= (\bar{U}u)_{V-A}(\bar{d}'b)_{V-A} + (\bar{U}c)_{V-A}(\bar{s}'b)_{V-A},
\end{aligned}
\tag{2}
$$

with the short notation $(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ and $U = u,\ c$. The primed fields are the weak interacting quarks, related to the mass eigenstates by the CKM mixing

$$
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad .
\tag{3}
$$

Let us observe that $Q_1^{Ub}$ corresponds to the same operator that we would have found in the Fermi pointlike approximation. In such an approximation, one has $C_1 = 1$, $C_2 = 0$. QCD corrections have both the effect of introducing a new operator, with the same flavor content, but a different color structure, and of modifying the Wilson coefficients.

We will need the general features of effective theories recalled in this section, in order to understand the relation between HQET and QCD, as well as to stress the difference arising in this situation where we just want to describe heavy quark dynamics and therefore cannot completely integrate them out.

## 3   Heavy Quark Effective Theory

The aim of this section is to give an intuitive picture of heavy quark symmetries and the basis of the systematic HQET formulation.[12]

Let us recall what is specifically meant by heavy quark. Except for the quark masses, QCD has a single adjustable parameter, $\Lambda_{QCD}$, which could be viewed as the scale separating the strong coupling regime from the weak one. It depends on the number of "active" flavors at a given scale. It cannot be precisely determined, but, roughly speaking, it can be considered as the inverse of the radius of a hadron, where confinement effects act to give the bound state. For $R_{had} \simeq 1$ fm, it turns out that $\Lambda_{QCD} \simeq 200$ MeV. Therefore we can classify heavy quarks as those with mass larger than this parameter. Since we expect that typical strong interactions inside the hadron involve exchange of gluons with a virtuality of order $\Lambda_{QCD}$, this gives the heavy quark a special role in the hadron, as we shall see.

We will therefore consider $c,\ b,\ t$ as heavy quarks, while $u,\ d,\ s$ will be considered light. However, as well as the strange quark cannot be always considered light in the application of chiral perturbation theory techniques, in the same sense the considerations which are derived within HQET should be applied with great caution to charm. Finally, the top quark cannot be considered at all, since it will decay before any bound state can be formed. Hence, strictly speaking, HQET is a theory of $b$ decays.

In order to describe qualitatively a system containing a heavy quark $Q$, we will consider the explicit case of a heavy meson, and we will refer to its light antiquark plus the cloud of gluons as the light degrees of freedom. We would like to consider this system in the large heavy quark mass limit, $m_Q \to \infty$. In this limit (which is is quite close to reality) the chromomagnetic interaction between the heavy quark spin $s_Q$ and the light degrees of freedom total angular momentum $s_\ell$ dies off – it is inversely proportional to the heavy quark mass (just like the familiar quantum mechanical spin-orbit interaction). Therefore such quantities are expected to decouple in the heavy quark limit. Besides, in the QCD Lagrangian the dependence on the quark flavor is only contained in the mass term, so we also expect that the heavy quark flavor becomes irrelevant for $m_Q \to \infty$. This is the intuitive content of the, by now, well-known heavy quark spin symmetry realized within HQET. This symmetry tells us that the light degrees of freedom in a heavy hadron are the same independently of the heavy quark spin and flavor quantum numbers.

One notes more than once the strict analogy with atomic physics, observing that the flavor symmetry could be compared to the fact that different isotopes of the same element have identical chemical properties, while the spin symmetry translates in atomic physics in the (almost) degeneracy of hyperfine levels. These observations already point out a limit of the considered framework, since they allow us to exploit the decoupling of the heavy quark in order to work out relations among various quantities involved in heavy hadron decays; on the other hand, they do not allow us to say anything about the light degrees of freedom.

Another result that can be intuitively understood is known in the literature as the velocity superselection rule, which is due to Georgi.[4] The momentum of a heavy hadron $H_Q$ can be written as $p_H^\mu = m_H \, v^\mu$, while for the heavy quark: $p_Q^\mu = m_Q \, v^\mu + k^\mu$, where $k$ is a residual momentum of $\mathcal{O}(\Lambda_{QCD})$. Since, on the other hand $p_Q^\mu = m_Q \, v_Q^\mu$, one finds that, for $m_Q \to \infty$, $v_Q^\mu = v^\mu$, i.e. strong interactions conserve the heavy quark velocity.

In order to give a systematic structure to these intuitive properties, we would like to build up an effective field theory for large $m_Q$. However, the procedure outlined in the previous paragraph is not very useful if we only want to describe heavy hadron interactions. To use a familiar example, if we were to describe processes where the $W$ boson appears as a final or initial state, we could no more use the effective Fermi Lagrangian where the $W$ is no more a dynamical field. For this reason, HQET is not obtained integrating out the heavy quark, but only the "small" components of its spinor. Therefore, we have to start from the QCD Lagrangian relative to the heavy quark

$$\mathcal{L}_Q = \bar{Q}(i \not{D} - m_Q)Q \tag{4}$$

where $D$ is the covariant derivative in the fundamental representation, and redefine the heavy quark spinor $Q$ as follows:

$$Q(x) = e^{-im_Q v \cdot x}[h_v(x) + H_v(x)] \ , \tag{5}$$

where $h_v$ and $H_v$ are defined by the application of the velocity projectors,

$$h_v = \frac{1 + \slashed{v}}{2}Q \qquad H_v = \frac{1 - \slashed{v}}{2}Q \ , \tag{6}$$

and therefore they satisfy: $\slashed{v} h_v = h_v$, $\slashed{v} H_v = -H_v$. Substituting Eq. (6) in Eq. (4) one can derive the equation of motion for $H_v$,

$$H_v = \frac{1}{iv \cdot D + 2m_Q}i \slashed{D}_\perp h_v \ , \tag{7}$$

where $D_\perp^\mu = D^\mu - v \cdot D v^\mu$. Equation (7) can be used to eliminate $H_v$ from the Lagrangian, leading to the following one, equivalent to $\mathcal{L}_Q$:

$$\mathcal{L} = \bar{h}_v iv \cdot Dh_v + \bar{h}_v i \slashed{D}_\perp \frac{1}{iv \cdot D + 2m_Q}i \slashed{D}_\perp h_v \ . \tag{8}$$

The second term can now be expanded in the powers of $1/m_Q$, yielding

$$\mathcal{L} = \bar{h}_v iv \cdot Dh_v + \frac{1}{2m_Q}\bar{h}_v(i \slashed{D}_\perp)^2 h_v + \frac{1}{2m_Q}\bar{h}_v \frac{g_s \sigma_{\alpha\beta} G^{\alpha\beta}}{2} h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)^2 \ . \tag{9}$$

The second term represents the kinetic energy of the heavy quark due to its residual momentum $k$. The last term stems from the chromomagnetic coupling of the heavy quark spin to the gluon field. The fact that this term appears only at the order $1/m_Q$ is the origin of the spin symmetry. At the leading order in the heavy quark expansion, we simply have

$$\mathcal{L}_{HQET} = \bar{h}_v iv \cdot Dh_v \ , \tag{10}$$

from which the following Feynman rule for the heavy quark propagator can be derived:

$$\frac{i}{v \cdot k}\frac{1 + \slashed{v}}{2} \ , \tag{11}$$

as well as the one for the quark-gluon vertex,

$$i \, g_s T^a v_\mu \ . \tag{12}$$

$T^a$ are the $SU(3)$ generators, $T^a = \lambda^a/2$, where $\lambda^a$ stand for the Gell-Mann matrices. Again, we recognize that spin symmetry holds from the absence of

gamma matrices in Eq. (12); besides, the independence of $\mathcal{L}_{HQET}$ of $m_Q$ signals the flavor symmetry. Most of the results that we will report in this chapter exploit the symmetries of $\mathcal{L}_{HQET}$. On the other hand, the predictive power of the approach could be improved assessing the role of symmetry-breaking terms, i.e. including subleading terms in the heavy quark expansion. This can be done starting from the result in Eq. (7), which gives us the possibility to write each QCD operator as an expansion in the powers of $1/m_Q$. In fact, expanding Eq. (7), one can derive the expression for the heavy quark field $Q$ in QCD at the order $1/m_Q$,

$$ Q(x) = e^{-im_Q v \cdot x} \left( 1 + \frac{i \not{D}_\perp}{2m_Q} \right) h_v \, , \tag{13} $$

from which we can build any desired QCD operator, for example a current, at the considered order. In practice we are interested in the matrix elements of such operators, which we would like to express by an expansion in $1/m_Q$. However, the states on which the operators act still contain a dependence on $m_Q$, since, at the next to leading order in the heavy quark expansion, $h_v$ does not satisfy the same equation of motion stemming from Eq. (10). The way out is to consider once and for all Eq. (10) as the HQET lagrangian; then $h_v$ exactly satisfies the equation $\not{v} h_v = h_v$. The subleading terms in Eq. (9) are treated as perturbations. In this way, as in the usual spirit of the perturbation theory, the results obtained in the presence of the perturbation can be expressed in terms of the unperturbated quantities.

As a concrete example of how an asymptotic relation could be modified by the inclusion of $1/m_Q$ corrections, let us consider the heavy hadron masses. On the basis of spin symmetry, one can argue that hadrons differing only in the orientation of the heavy quark spin should be degenerate in mass. For example, this should hold in the case of the pseudoscalar and vector heavy flavored mesons. In the charm case the experimental values give: $m_{D^*} - m_D \simeq 140$ MeV, while for the beauty we get: $m_{B^*} - m_B \simeq 47$ MeV. This is not surprising, since we expect heavy quark symmetry to work better in the $b$ case. However, we can go further by predicting the size of the corrections: if these are $\mathcal{O}\left(1/m_Q\right)$, one should have

$$ m_{D^*}^2 - m_D^2 \simeq m_{B^*}^2 - m_B^2 \, . $$

Experimentally this is quite well confirmed,

$$ m_{D^*}^2 - m_D^2 \simeq 0.49 \text{ GeV}^2 \, , \quad m_{B^*}^2 - m_B^2 \simeq 0.55 \text{ GeV}^2 \, . $$

The previous considerations allow us to express the $1/m_Q$ corrections in terms of the matrix elements of the perturbations appearing in Eq. (9). Let us define

the expectation values of the two subleading terms in Eq. (9) on a hadron $H_Q$ as follows:

$$\mu_\pi^2 = \frac{1}{2m_Q} \langle H_Q | \bar{h}_v (i\, \slashed{D}_\perp)^2 h_v | H_Q \rangle , \qquad (14)$$

$$\mu_G^2 = \frac{1}{2m_Q} \langle H_Q | \bar{h}_v \frac{g_s \sigma_{\alpha\beta} G^{\alpha\beta}}{2} h_v | H_Q \rangle . \qquad (15)$$

Notice that the states are normalized to $2m_Q$, so that the previous quantities are independent of $m_Q$. Using this notation, the mass of $H_Q$ can be written as follows:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2}\right) . \qquad (16)$$

Since the chromomagnetic operator is responsible for the spin symmetry breaking, and hence for the mass difference of the hadrons which differ only in the orientation of the heavy quark spin, it is possible to relate $\mu_G^2$ to such a difference. In particular, one can write $\mu_G^2 = -2 \left[ J(J+1) - \frac{3}{2} \right] \lambda_2$, where $\lambda_2$, as well as $\bar{\Lambda}$ and $\mu_\pi^2$, does not depend on $m_Q$. From the measured mass splitting in the beauty mesons, where we expect our considerations to hold more accurately, it is possible to predict

$$\lambda_2 = \frac{1}{4}(M_{B^*}^2 - M_B^2) \simeq 0.12 \text{ GeV}^2 .$$

On the other hand, in the case of $B$ mesons, different controversial determinations of $\mu_\pi^2$ exist in literature,[13] mainly due to the difficulty of extracting this parameter from experimental data. Finally, $\bar{\Lambda}$ has the role of the mass difference between the heavy hadron and the heavy quark in the $m_Q \to \infty$ limit. It is of $\mathcal{O}(\Lambda_{QCD})$.

Let us observe that the relation $Q(x) = e^{-im_Q v \cdot x} h_v(x)$, stemming from Eq. (13) in the heavy quark limit, *defines* the heavy quark mass $m_Q$. This turns out to be a physical parameter, given by the relation $m_Q = M_{H_Q} - \bar{\Lambda}$, in the same limit. For example, the form factors describing the decay $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ depend on $\bar{\Lambda} = M_{\Lambda_b} - m_b$;[14] therefore, one could use this decay to determine $\bar{\Lambda}$, so that the experimental value of $M_{\Lambda_b}$ would provide a determination of $m_b$. In this sense, the mass appearing in Eq. (16) can be considered as the nonperturbative analog of the pole mass, which, despite being a well defined quantity in perturbation theory, suffers from renormalon ambiguities beyond it.[15] We shall see later in the discussion of inclusive decays that the adoption of such definition leads to the absence of $1/m_Q$ corrections to the partonic prediction.

### 3.1  Weak Decays of Heavy Mesons

An important role is played by heavy quark spin-flavor symmetries when considering weak decays of heavy mesons. Let us consider for example the elastic scattering of a pseudoscalar meson $P(v)$ induced by an external vector current coupled to the heavy quark. Since in the $m_Q \to \infty$ limit the light degrees of freedom are decoupled from the heavy quark, it looks sensible to describe this process as if the current were acting only on the heavy quark. We can write

$$\langle P(v')|\bar{h}_{v'}\gamma^\mu h_v|P(v)\rangle = \langle Q, v', s'_Q|\bar{h}_v\gamma^\mu h_v|Q, v, s_Q\rangle \, \langle L, s'_\ell|L, s_\ell\rangle \qquad (17)$$

where $L$ represents the light degrees of freedom. The content of Eq. (17) amounts to a *factorization*, where the last term represents the overlap of the light degrees of freedom. If $v = v'$, spin symmetry assures us that nothing is changed for the light degrees of freedom: the overlap is simply $\delta_{s_\ell,s'_\ell}$. This holds also if the heavy quark in the final state is a different one, thanks to the flavor symmetry.

If $v \neq v'$, the decoupling of the light degrees of freedom still allows us to write such a factorization. However, now the light degrees of freedom in the final state are in general changed with respect to the initial state and the overlap will be a new function $\xi(v \cdot v')$ not depending on the spin and the flavor of the heavy quark, and which is normalized to 1 for $v \cdot v' = 1$, since the zero component of the vector current is the generator of the flavor symmetry. A detailed proof of such normalization will be given later. This function is usually referred to as the Isgur-Wise universal form factor.[2,16] Its universality is apparent if we consider that it is possible to describe the semileptonic decays $B \to D$ or $B \to D^*$ in terms of this single function, with the result of a great simplification of the theoretical study of these processes, which are usually described by 2 and 4 form factors, respectively. The possibility to give asymptotic relations between these form factors is a quite appealing feature. As a matter of fact, one could consider such relations as a test for models predicting these quantities or for the outcome of nonperturbative techniques, such as lattice or QCD sum rules. For example, Neubert has shown that not one of the commonly used quark models proposed in the literature satisfied the asymptotic behaviour predicted by the heavy quark symmetry.[12]

Interestingly enough, the argument that lead us to the introduction of the Isgur-Wise function can be repeated in similar situations. Spin symmetry tells us that for each value of the light degrees of freedom total angular momentum $s_\ell$ there are two degenerate states with total spin: $\vec{J} = \vec{s}_Q + \vec{s}_\ell$, according to the rules of addition of angular momentum. These ideally degenerate states will conveniently fit into doublets. Besides, $\vec{s}_\ell$ can be written as the sum of

the light antiquark spin $\vec{s}_q$ plus the orbital angular momentum $\vec{\ell}$ of the light degrees of freedom with respect to the heavy quark. Therefore, a given doublet could be identified by the value of $s_\ell$ and the parity of the states $P = (-1)^{\ell+1}$. If $\ell = 0$, the two degenerate states form the doublet of pseudoscalar and vector mesons $(P, P^*)$ (with $P = D$, $B$); these have $J^P_{s_\ell} = (0^-, 1^-)_{1/2}$. Therefore the Isgur-Wise function is the form factor describing the decay of an element of this doublet to another element of the same doublet.

Let us consider the case $\ell = 1$. Now it could be either $s_\ell = 1/2$ or $s_\ell = 3/2$. The two corresponding doublets have positive parity; they are: $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$, $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$. We will refer to elements of these doublets generically as $(P_0, P'_1)$ and $(P_1, P_2)$, respectively. The charmed $2^+_{3/2}$ state has been experimentally observed and denoted as the $D^*_2(2460)$ meson, with $m_{D^*_2} = 2458.9 \pm 2.0$ MeV, $\Gamma_{D^*_2} = 23 \pm 5$ MeV and $m_{D^*_2} = 2459 \pm 4$ MeV, $\Gamma_{D^*_2} = 25^{+8}_{-7}$ MeV for the neutral and charged states, respectively.[17] The HQET state $1^+_{3/2}$ can be identified with $D_1(2420)$, with $m_{D_1} = 2422.2 \pm 1.8$ MeV and $\Gamma_{D_1} = 18.9^{+4.6}_{-3.5}$ MeV,[17] even though a $1^+_{1/2}$ component can be contained in such physical state due to the mixing allowed for the finite value of the charm quark mass. Actually, another state with $J^P = 1^+$ has been identified [18] with mass $m_{D^{*0}_1} = (2461 \pm^{41}_{34} \pm 10 \pm 32)$ MeV. Both the states $2^+_{3/2}$ and $1^+_{3/2}$ decay to hadrons by $d-$wave transitions, which explains their narrow width; the strong coupling constant governing their two-body decays can be determined using experimental information.[19] On the other hand, the strong decays of the states belonging to the $s^P_\ell = \frac{1}{2}^+$ doublet $(D_0, D'_1)$ occur through $s$-wave transitions, with larger expected widths than in the case of the doublet $\frac{3}{2}^+$. Indeed, analyses of the coupling constant governing the two-body hadronic transitions in QCD sum rules predict $\Gamma(D^0_0 \to D^+ \pi^-) \simeq 180$ MeV and $\Gamma(D'^0_1 \to D^{*+} \pi^-) \simeq 165$ MeV.[20] Estimates [20,21] of the mixing angle $\alpha$ between $D^*_1$ and $D_1$ give $\alpha \simeq 16^0$. Also in the case of $D_s$ and $B$ mesons experimental evidences of these states have been reported.[22,23]

Again we can introduce universal form factors, one for each transition between couples of doublets. The universal function describing the transition of an element of the fundamental doublet $(P, P^*)$ to the doublet $(P_0, P'_1)$ with $s_\ell = 1/2$ will be referred to as $\tau_{1/2}(v \cdot v')$, while the transitions of the fundamental doublet to the $(P_1, P_2)$ one with $s_\ell = 3/2$ will be described by the function $\tau_{3/2}(v \cdot v')$. However, the heavy quark symmetry does not predict the normalization of these functions, though a great simplification is again obtained, since the two $\tau$ functions take the place of 14 form factors, usually employed to describe these transitions.

We can stress again what has been said in the beginning: HQET is very

powerful in providing us with relations among different quantities entering in the description of heavy hadron processes, and the universal form factors are an important example. However, these quantities cannot be computed within the same context, since they represent the overlap of the light degrees of freedom. They are essentially nonperturbative objects, and should be computed by some other means. We shall see that they will play a role in the subsequent analyses.

A very useful way to derive the structure of the matrix elements in HQET is to combine the members of a given doublet in a single wave function. [24] For example, for the fundamental doublet one can write

$$H_a = \frac{1 + \not{v}}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5] \ , \tag{18}$$

where the first term represents the vector meson, the second the pseudoscalar one, and $a$ is a light flavor index. The operators $P^*_{a\mu}$, $P_a$ destroy a vector and a pseudoscalar meson, respectively, and contain a factor $\sqrt{m_P}$. Besides, $v^\mu P^*_{a\mu} = 0$. Explicitly one can write

$$
\begin{aligned}
P(v) &= -\sqrt{m_P}\frac{1 + \not{v}}{2}\gamma_5 \ , \\
P(v, \epsilon) &= \sqrt{m_P}\frac{1 + \not{v}}{2}\not{\epsilon} \ .
\end{aligned}
\tag{19}
$$

Finally, $\bar{H}_a = \gamma_0 H_a^\dagger \gamma_0$. Let us now consider the transition between two heavy mesons $P(v) \to P'(v')$ induced by an external current coupled to the heavy quarks, $J = h_{v'}^{Q'}\Gamma h_v^Q$, where $\Gamma$ is a product of Dirac matrices appropriate to the process at hand. Taking into account the previous considerations on the factorization of the overlap of the light degrees of freedom, it is possible to derive the most general decomposition for the transition matrix element, satisfying all the symmetry requirements (Lorentz covariance, heavy quark symmetry and parity),

$$\langle P'(v')|J|P(v)\rangle = Tr[\Xi(v, v')\bar{P}'(v')\Gamma P(v)] \ , \tag{20}$$

where $\Xi(v, v')$ is a matrix containing the overlap of the light degrees of freedom as well as the right tensorial structure to contract free indices in the meson wave functions in order to reproduce the correct Lorentz structure of the matrix element. In the case of $B \to D^{(*)}$ semileptonic transitions, one has $\Xi(v, v') = -\xi(v \cdot v')$, and the following matrix elements can be worked out,

$$\langle D(v')|\bar{h}_{v'}^c\gamma^\mu h_v^b|\bar{B}(v)\rangle = \xi(v \cdot v') \sqrt{m_D m_B} (v^\mu + v'^\mu) \ , \tag{21}$$

and

$$\langle D^*(\epsilon', v') | \bar{h}^c_{v'} \gamma^\mu (1 - \gamma_5) h^b_v | \bar{B}(v) \rangle$$

$$= i\xi(v \cdot v') \sqrt{m_D m_B}$$

$$\times \quad [\epsilon^{\nu\alpha\mu\beta} \epsilon'^*_\nu v'_\alpha v_\beta + (1 + v \cdot v') \epsilon'^{*\mu} - \epsilon'^{*\nu} v_\nu v'^\mu]. \tag{22}$$

The two positive parity doublets introduced before can be represented analogously to Eq. (18). The $s_\ell = 1/2$ doublet is described by the following matrix:

$$S_a = \frac{1 + \not{v}}{2} (P_1'^\mu \gamma_\mu \gamma_5 - P_0) \quad, \tag{23}$$

while the $s_\ell = 3/2$ one is given by

$$T_a^\mu = \frac{1 + \not{v}}{2} \left\{ D_2^{\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} D_1^\nu \gamma_5 \left[ g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - v^\mu) \right] \right\} \quad. \tag{24}$$

The constructions could be further extended to higher spins.[25] Let us now demonstrate in detail the normalization of the Isgur-Wise function, since it will play an important role in the sequel. As it can easily be checked, for equal velocities, the vector current $\bar{h}_v \gamma_\mu h_v$ in the effective theory is conserved. Therefore, on account of Noether's theorem, there is a conserved charge, $N = \int d^3x h_v^\dagger(x) h_v(x)$. At zero recoil, using invariance under translations, one has

$$\int d^3x \langle P(v) | \bar{h}_v^\dagger(x) h_v(x) | P(v) \rangle = \int d^3x \langle P(v) | \bar{h}_v^\dagger(0) h_v(0) | P(v) \rangle \quad. \tag{25}$$

The operator $\bar{h}_v^\dagger h_v$ is the "number" operator, which counts the number of heavy quarks. Therefore, we have

$$\int d^3x \langle P(v) | \bar{h}_v^\dagger(0) h_v(0) | P(v) \rangle = V \langle P(v) | P(v) \rangle = V \, 2M_P v^0 \quad, \tag{26}$$

where $V$ is a normalization volume. On the other hand, using Eq. (21), it holds also

$$\int d^3x \langle P(v) | \bar{h}_v^\dagger(x) h_v(x) | P(v) \rangle = V \, \xi(1) \, 2M_P v^0 \quad. \tag{27}$$

Comparing Eqs. (26) – (27), it follows that

$$\xi(1) = 1 \, . \tag{28}$$

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Symmetry arguments have been used in physics more than once in order to derive interesting phenomenological quantities. For example, the value of the vector coupling $g_V = 1$ in the nuclear beta decay of the neutron, which follows from assuming exact strong isospin invariance of nuclear interactions, has allowed to precisely extract $V_{ud}$ from such a process. Moreover, in the limit of exact $SU(3)_F$ symmetry, the form factor describing the semileptonic decay $K \to \pi \ell \nu_\ell$, is normalized at zero recoil, receiving corrections only at the the second order in the symmetry breaking parameter $m_s - m_u$. [26] This result, together with the analysis of such $\mathcal{O}(m_s - m_u)^2$ corrections,[27] has given the possibility to obtain an accurate determination of $V_{us}$. We shall see that the normalization of the Isgur-Wise function will play an analogous role in the extraction of $V_{cb}$ from experimental data.

### 3.2 Matching

All our previous considerations have been derived in the heavy quark limit, $m_Q \to \infty$. For energies much below $m_Q$ we could assume that HQET represents a good approximation to reality, in the same sense that Fermi theory does for energies much below $M_W$. However, what should be supplied in both cases are short distance corrections, which, as already noticed, provide new operators and modify the Wilson coefficients. In this way we reach an important goal: we can exploit HQET in the long distance regime, where we do not know how to cope with QCD; on the other hand, the corrections to the asymptotic predictions can be computed sistematically in perturbative QCD.

It can be easily understood why the ultraviolet behaviour of the two theories is different: the typical terms arising in short distance computations involve the logarithms of the heavy quark mass, exactly as it was for the $W$, and therefore these logs diverge when taking the $m_Q \to \infty$ limit. This is particularly important when one considers those operators, such as the vector or the axial-vector current, which do not require renormalization in QCD. In fact, these operators do require renormalization in HQET.

Let us consider one such operator, $J_{QCD}$. We have seen that it can be expressed in terms of operators in the effective theory, which arrange themselves in an expansion in the inverse $m_Q$ powers,

$$J_{QCD} = J_{HQET}^{(0)} + \frac{1}{m_Q} J_{HQET}^{(1)} + \dots \ ; \tag{29}$$

this equality should be understood in the sense of matrix elements, since, according to the discussion in Sec. 3, the states on which they act are different, those appearing in the matrix element of the operator on the left-hand side being dependent on $m_Q$, on the contrary of those on which the operators on the

right act. Each term of the expansion receives further short distance corrections. As a consequence, at each order in $1/m_Q$, several operators contribute to Eq. (29), multiplied by suitable Wilson coefficients, so that each $J^{(i)}$ in such equation will become $J^{(i)} = \sum_j C_j(\mu) \, J_j^{(i)}(\mu)$. Let us focus our attention just on the first term in Eq. (29), and drop for simplicity the superscript 0 and the subscript $HQET$, where from now on the operators without indices will refer to HQET. Equation (29) becomes

$$J_{QCD} = \sum_i C_i(\mu) \, J_i(\mu) + \mathcal{O}\left(\frac{1}{m_Q}\right) = \sum_i C_i(\mu) \, Z_{ij}^{-1}(\mu) \, J_j^{bare} + \mathcal{O}\left(\frac{1}{m_Q}\right) \ ,$$
(30)

where we took into account the fact that HQET currents require renormalization by introducing a matrix of renormalization constants $Z_{ij}$. This is a matrix because in general we could have different operators with the same quantum numbers contributing at each order, which could mix under renormalization. Since both $J_{QCD}$ and $J^{bare}$ are $\mu$-independent quantities, the scale dependence should cancel between $C_i(\mu)$ and $Z_{ij}(\mu)$. This gives rise to the renormalization group equation for the Wilson coefficients

$$\mu\left(\frac{d}{d\mu} - \hat{\gamma}^T\right) C(\mu) = 0 \ ,$$
(31)

where the coefficients have been collected in a single vector $C$, and $\hat{\gamma}^T$ is the transpose of the matrix of anomalous dimensions, obtained by the matrix $\hat{Z}$ of the renormalization constants,

$$\hat{\gamma} = \hat{Z}^{-1}\mu\frac{d}{d\mu}\hat{Z} \ .$$
(32)

At a suitable high scale $m \simeq \mathcal{O}(m_Q)$, one can obtain the Wilson coefficients by comparing the diagrams including gluon radiative corrections in the full and in the effective theory at the desired order in $\alpha_s$, since in this case there are no more large logs and usual perturbation theory works well. In this way, one can write the solution of Eq. (31) as follows:

$$C(\mu) = U(\mu, m)C(m) \ .$$
(33)

Let us consider this solution in the case without mixing, so that $\gamma$ is a number. This is a relevant example because, as we shall see later, this is just the case of the vector and axial vector currents, where no mixing occurs, and hence the anomalous dimension is a diagonal matrix, allowing us to apply the

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considerations which follow. In this situation, one obtains

$$U(\mu, m) = \left[ \frac{\alpha_s(m)}{\alpha_s(\mu)} \right]^{\frac{\gamma_0}{2\beta_0}} \left[ 1 + \frac{\alpha_s(m) - \alpha_s(\mu)}{4\pi} S + \dots \right] \ , \tag{34}$$

where

$$S = \frac{\gamma_1 \ \beta_0 - \beta_1 \ \gamma_0}{2\beta_0^2} \ , \tag{35}$$

and $\gamma_0$, $\gamma_1$ and $\beta_0, \beta_1$ are the first two coefficients of the perturbative expansion of the anomalous dimension $\gamma$ and of the QCD $\beta$-function. The approximation of considering only the first factor in Eq. (34) corresponds to the leading log approximation, in which the large logs have been resummed. Introducing the one loop expression of $\alpha_s$ and expanding again this term, one recovers the perturbation theory result. The second term is the next-to-leading log correction, which requires the knowledge of the second coefficient of the anomalous dimension.

If $C(m)$ has been computed at $\mathcal{O}(\alpha_s)$,

$$C(m) = C_0 + C_1 \frac{\alpha_s}{4\pi} \ ,$$

it is possible to write the solution in Eq. (33) separating the $\mu$ dependence and the heavy mass dependence,

$$C(\mu) = \hat{C}(m)\hat{K}(\mu) \ , \tag{36}$$

where

$$\begin{aligned} \hat{C}(m) &= \left[ \alpha_s(m) \right]^{\gamma_0/2\beta_0} \left[ C_0 + (C_0 \ S + C_1) \frac{\alpha_s(m)}{4\pi} \right] \ , \\ \hat{K}(\mu) &= \left[ \alpha_s(\mu) \right]^{-\gamma_0/2\beta_0} \left( 1 - \frac{\alpha_s(m)}{4\pi} \ S \right) \ . \end{aligned} \tag{37}$$

We shall see in the following that this formalism allows a precise study of the heavy hadron transitions, with relevant phenomenological consequences.

## 4   Leptonic Decay Modes and Heavy Meson Leptonic Constants

The leptonic decay constants are non perturbative parameters which enter universally in the description of bound state effects in a given particle. For a pseudoscalar and a vector particle, respectively, they are defined by the matrix elements

$$\begin{aligned} \langle 0|A_\mu|P(p)\rangle &= if_P p_\mu \ , \\ \langle 0|V_\mu|P^*(p,\lambda)\rangle &= if_{P^*} m_{P^*} \epsilon_\mu(p,\lambda) \ , \end{aligned}$$

where $\epsilon$ is the polarization vector of the $P^*$ meson. The leptonic constant $f_P$ allows the complete description of the purely leptonic decay mode of the pseudoscalar meson, with the decay rate given by:

$$\Gamma(P \to \ell^- \bar{\nu}_\ell) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 \left(\frac{m_\ell}{m_P}\right)^2 m_P^3 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 \quad . \tag{38}$$

Therefore, the leptonic decay modes are the best places where to experimentally measure such hadronic parameters; indeed, in the case of pion the constant $f_\pi$ is precisely determined from the process $\pi^- \to \mu^- \bar{\nu}_\mu$. However, in the case of heavy mesons like the $B$ meson, things are hard. Purely leptonic decay channels present the difficulty of the helicity suppression factor

$$\left(\frac{m_\ell}{m_B}\right)^2 ,$$

that makes the purely leptonic decay mode hardly accessible in the electron and in the muon case (branching fractions of order $10^{-10}$, $10^{-7}$, respectively). In the case of the $\tau$ lepton, the helicity suppression is absent, but the experimental $\tau$ reconstruction is a difficult task.

One could hardly overemphasize the importance of the measurement of $f_B$, due to the role played by this parameter in the description of CP violation in the neutral $B$ system. Besides, as stems from Eq. (38), $f_B$ often appears together with some CKM matrix element, so that the extraction of either the decay constant or the relevant CKM element is affected by the uncertainty in the other parameter. This is especially important in the case of $V_{ub}$, which is still affected by quite a large uncertainty.

Adopting arguments inspired by HQET, other possibilities have been proposed, which avoid helicity suppression.[28] The arguments are based on the quark scaling behaviour of the pseudoscalar and the vector $\bar{q}Q$ decay constants. As a matter of fact, using the trace formalism as well as Eq. (18), one can show, in a straightforward manner, that in the limit of large $b$ quark mass

$$f_B = f_{B^*} = \frac{F}{\sqrt{m_B}} \quad . \tag{39}$$

The constant $F$ is another example of a universal parameter in HQET: in the heavy quark limit, it describes both pseudoscalar and vector heavy meson decays, irrespective of their flavor. The simple relation in Eq. (39) receives both short distance and $\mathcal{O}(m_Q^{-1})$ corrections; in the leading log approximation, one finds[6,29,30,31]

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{6/25} \tag{40}$$

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and

$$\frac{f_{P^*}}{f_P} = 1 - \frac{2\alpha_s(m_Q)}{3\pi} \quad , \tag{41}$$

with $P = D, \ B$. This observation inspired the proposal [28] of using radiative leptonic $B$ decays $B \rightarrow \ell\nu\gamma$, in addition to purely leptonic modes, to get information on $F$ and $f_B$. As a matter of fact, $B \rightarrow \ell\nu\gamma$ processes are not helicity suppressed, and their branching ratio, in the case of electron and muons in the final state, is estimated to be one order of magnitude larger than $\Gamma(B \rightarrow \mu\nu)$. A determination of $F$ by this mode, although affected by some amount of theoretical error, would indeed be very precious for the whole $B$ meson phenomenology.

## 5  Isgur-Wise Function $\xi(y)$ and the Determination of $|V_{cb}|$

Let us now consider the exclusive decays $B \rightarrow D^{(*)}\ell\nu$. The weak matrix elements relevant to these transitions can be conveniently parametrized in terms of form factors,

$$\frac{\langle D(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_D}} = h_+(y) \ (v + v')_\mu + h_-(y) \ (v - v')_\mu \ ,$$

$$\frac{\langle D^*(v', \epsilon)|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i \ h_V(y) \ \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta \ , \tag{42}$$

$$\frac{\langle D^*(v', \epsilon)|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(y)(y + 1)\epsilon_\mu^* - \left[h_{A_2}(y)v_\mu + h_{A_3}(y)v'_\mu\right]\epsilon^* \cdot v \ ,$$

where $v$ and $v'$ are the initial and final meson four-velocities, respectively, and $y = v \cdot v'$. These are the form factors that, in the $m_Q \rightarrow \infty$ limit, can all be expressed in terms of a universal function $\xi(y)$ introduced in Sec. 2. Comparing Eq. (42) with Eqs. (21) – (22), one finds

$$h_+(y) = h_V(y) = h_{A_1}(y) = h_{A_3}(y) = \xi(y) \ ,$$

$$h_-(y) = h_{A_2}(y) = 0 \ . \tag{43}$$

Considering short distance corrections, all the form factors are related to a single function in the following, more complicated way:

$$h_+(y) = \left\{C_1(y, \mu) + \frac{y + 1}{2} \left[C_2(y, \mu) + C_3(y, \mu)\right]\right\} \ \xi(y, \mu) \ ,$$

$$h_-(y) = \frac{y + 1}{2} \left[C_2(y, \mu) - C_3(y, \mu)\right] \ \xi(y, \mu) \ ,$$

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$$
\begin{aligned}
h_V(y) &= C_1(y, \mu)\, \xi(y, \mu)\,, \\
h_{A_1}(y) &= C_1^5(y, \mu)\, \xi(y, \mu)\,, \\
h_{A_2}(y) &= C_2^5(y, \mu)\, \xi(y, \mu)\,, \\
h_{A_3}(y) &= \left[ C_1^5(y, \mu) + C_3^5(y, \mu) \right]\, \xi(y, \mu)\,.
\end{aligned}
\tag{44}
$$

In the previous equations, $C_i^{(5)}(y, \mu)$ represent the short distance coefficients connecting the QCD vector (axial-vector) current to the corresponding HQET current; there are three such coefficients because now we have three possible structures, i.e. $\gamma_\mu$, $v_\mu$, $v'_\mu$. The $\mu$-dependence is the same for all the functions $C_i^{(5)}$; therefore, according to Eq. (36), one can extract such a dependence by writing [12]

$$
C_i^{(5)}(y, \mu) = \hat{C}_i^{(5)}(m_b, m_c, y)\, K_{hh}(y, \mu)
\tag{45}
$$

where

$$
K_{hh} = [\alpha_s(\mu)]^{-a_{hh}(y)} \left\{ 1 - \frac{\alpha_s(\mu)}{\pi} Z_{hh}(y) \right\},
$$

with $a_{hh} = \frac{2}{9}\gamma(y)$, and $\gamma(y)$ is related to the velocity-dependent anomalous dimension of the heavy-heavy current in HQET[32]

$$
\begin{aligned}
\gamma(y) &= \frac{4}{3}[yr(y) - 1]\,, \\
r(y) &= \frac{\ln\left( y + \sqrt{y^2 - 1} \right)}{\sqrt{y^2 - 1}}\,.
\end{aligned}
\tag{46}
$$

As for the coefficient $Z_{hh}$, we refer the reader to the original literature where it has been derived. [12,33,31,34] The expressions of the coefficient functions $\hat{C}_i^{(5)}$ are known at the next-to-leading order;[35,12] it the leading-log approximation they simply read

$$
\hat{C}_1 = \hat{C}_1^5 = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{a_I} [\alpha_s(m_c)]^{a_{hh}}\,,
$$

with $a_I = -\frac{6}{25}$, the coefficients $\hat{C}_2^{(5)}$ and $\hat{C}_3^{(5)}$ being zero.

The $\mu$-dependence of the coefficients $C_i^{(5)}$ should cancel against the function $\xi(y, \mu)$, so that the form factor

$$
\xi_{ren}(y) = K_{hh}(y, \mu)\xi(y, \mu)
\tag{47}
$$

is a renormalization group invariant function. Notice that the pattern of radiative corrections do not modify the normalization $\xi(1) = 1$. To quantify

this statement, let us introduce two combinations of short distance coefficients which will be relevant in the following discussion,

$$\eta_V = \sum_1^3 C_i(y=1), \qquad \eta_A = C_1^5(y=1) \ . \tag{48}$$

For the $b \to c$ transition, the explicit calculation[35] gives

$$\eta_V = 1.025 \pm 0.006, \quad \eta_A = 0.986 \pm 0.006 \ . \tag{49}$$

However, in the case of flavor conserving heavy quark currents, one has

$$\eta_V = 1, \quad \eta_A = 1 - \frac{2\alpha_s(m_Q)}{3\pi} \ , \tag{50}$$

which explicitly shows that the flavor conserving vector current is not renormalized, assuring the $\xi$ normalization at zero recoil point.

$\mathcal{O}\left(1/m_Q\right)$ corrections modify this picture, introducing new operators in the expansion of the QCD currents in terms of the HQET ones. However, the matrix elements of these new operators all vanish in the zero recoil point at this order, so that the normalization of the matrix elements in Eq. (42) is further preserved, receiving corrections only at order $1/m_Q^2$. This is the content of Luke's theorem.[36] Such a theorem assures that the two form factors $h_+$, $h_{A_1}$ in Eq. (42) receive only second order corrections in the inverse heavy quark mass expansion at zero recoil point,

$$
\begin{aligned}
h_V(1) &= \eta_V + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \ , \\
h_{A_1}(1) &= \eta_A + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \ .
\end{aligned}
\tag{51}
$$

However, this does not hold for all the form factors appearing in Eq. (42), which simply do not contribute at $v = v'$ because they are multiplied by vanishing kinematical factors.[37]

This analysis is particularly relevant since the exclusive decays $B \to D^{(*)} \ell \nu$ allow to access the CKM matrix element $V_{cb}$. However, due to the explained pattern of corrections, only the decay to the charmed vector meson is suitable, since in this case the rate, near the zero recoil point, is proportional to $|h_{A_1}(y)|^2$, which is protected by Luke's theorem. This is not the case for the transition to the $D$ meson.[38]

The differential decay rate for the process $B \to D^* \ell \nu$ can be written as follows:

$$
\frac{d\Gamma}{dy}(B \to D^* \ell \nu) = \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{y^2 - 1}(y+1)^2 \tag{52}
$$

$$
\times \left[1 + \frac{4y}{y+1} \frac{m_B^2 - 2y m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right] |V_{cb}|^2 \mathcal{F}^2(y) \; ,
$$

where the function $\mathcal{F}^2(y)$ differs from the Isgur-Wise function for short distance and $\mathcal{O}(m_Q^{-1})$ corrections. What should be noticed is that: *i*) the normalization of $\mathcal{F}$ at $y = 1$ is known; *ii*) the spectrum vanishes at $y = 1$. The comparison of the experimental data with the previous result could be translated in a result for the product $|V_{cb}|^2 \mathcal{F}(1)$, after a suitable extrapolation to the zero recoil point due to the vanishing of the spectrum. This has lead to an accurate determination of $|V_{cb}|$, which can be considered among the most important applications of the heavy quark symmetries. We quote the last result:[39]

$$
|V_{cb}| = (39.8 \pm 1.8 \pm 2.2) \times 10^{-3} \; . \tag{53}
$$

## 6 Determination of the Universal Form Factors from the QCD Sum Rules

HQET allows one to derive model independent relations, holding in the heavy quark limit, mainly on the basis of the decoupling of the light degrees of freedom from the spin of the heavy quark. We have noticed that in such a limit it is possible to introduce a whole series of nonperturbative parameters with a universal meaning, such as the decay constant $F$, or the universal form factors describing weak decays among the members of doublets of asymptotically degenerate states. The usefulness of such relations is evident in the case of the Isgur-Wise function, where one can use symmetry arguments in order to predict the zero recoil point normalization and can use this information to extract $V_{cb}$ from experimental data. However, we often need something more, i.e. the quantitative estimate of these universal quantities, which cannot be done within HQET, but requires the use of some nonperturbative technique, such as QCD sum rules or lattice QCD. In particular, the use of QCD sum rules in the framework of HQET revealed a very powerful tool and many HQET parameters have been estimated in this context. We shall review in the following subsections the determination of the Isgur-Wise function $\xi$ [40,33,41] and the computation of the analogous universal form factors $\tau_{1/2}$, $\tau_{3/2}$, describing the transitions of a $B$ meson to excited positive parity charmed mesons.[42,43]

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As we shall see, a preliminary part of such computations is the determination, within the same QCD sum rule framework, of the leptonic constants of the heavy mesons. The functions $\xi$ and $\tau_{1/2}$ have been computed including $\mathcal{O}(\alpha_s)$ corrections, so that we shall have the chance to discuss the role of such corrections in practical situations.

Let us summarize the basic derivation of a generic universal form factor using QCD sum rules. The starting point is a three-point correlator, defined in the effective theory,

$$
\begin{aligned}
\Pi_{1,2}(\omega, \omega', y) &= i^2 \int dx\, dz\, e^{i(k'x - kz)} \langle 0 | T[J_1^{v'}(x) J_W(0) J_2^v(z)^\dagger] | 0 \rangle \\
&= \Pi(\omega, \omega', y)\, Tr\left\{ \Gamma_1 \frac{1 + \slashed{v}'}{2} \Gamma \frac{1 + \slashed{v}}{2} \Gamma_2^\dagger \right\}
\end{aligned}
\tag{54}
$$

where $J_1^{v'} = \bar{q}\Gamma_1 h_{v'}^{Q'}$, $J_2^v = \bar{q}\Gamma_2 h_v^Q$ are the effective currents interpolating the heavy mesons with heavy quarks $Q'$ and $Q$ respectively.[a] Moreover, $J_W = h_{v'}^{Q'} \Gamma h_v^Q$ is the weak current corresponding to the transition $h_v^Q \rightarrow h_{v'}^{Q'}$. The variables $k, k'$ are the residual momenta, obtained by the expansion of the heavy meson momenta in terms of the four-velocities: $P = m_Q v + k$, $P' = m_{Q'} v' + k'$.

Using the analyticity of $\Pi(\omega, \omega', y)$ in the variables $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$ at fixed $y$, one can represent the correlator (54) by a double dispersion relation of the form

$$
\Pi(\omega, \omega', y) = \int d\nu d\nu' \frac{\rho(\nu, \nu', y)}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} \quad ,
\tag{55}
$$

apart from possible subtraction terms. The correlator $\Pi(\omega, \omega', y)$ receives contributions from poles located at positive real values of $\omega$ and $\omega'$, corresponding to the physical single particle hadronic states in the spectral function $\rho(\nu, \nu', y)$.

This contribution is proportional to the universal function $\Xi$ appropriate to the transition at hand, through the following relation:

$$
\Pi_{pole}(\omega, \omega', y) \propto \frac{\Xi(y, \mu) F_1(\mu) F_2(\mu)}{(2\bar{\Lambda}_1 - \omega - i\epsilon)(2\bar{\Lambda}_2 - \omega' - i\epsilon)} \quad ,
\tag{56}
$$

where $\mu$ is the renormalization scale and $F_1(\mu)$, $F_2(\mu)$ are the effective leptonic constants of the heavy mesons interpolated by the currents $J_1^{v'}$ and $J_2^v$ respectively, in analogy to Eq. (39). The mass parameters $\bar{\Lambda}_1$ and $\bar{\Lambda}_2$ identify the

---

[a]The issue of the choice of the interpolating currents in HQET is extensively discussed by Dai *et al.*[44]

position of the poles in $\omega$ and $\omega'$, and can be interpreted as binding energies of the heavy mesons, in accordance with Eq. (16).

The higher state contributions to $\rho(\nu, \nu', y)$ can be taken into account by a QCD continuum starting at some thresholds $\nu_c$ and $\nu'_c$, and are modeled by the perturbative spectral function $\rho^{pert}(\nu, \nu', y)$ according to the quark-hadron duality assumption.[45] Here, $\rho^{pert}$ is the absorptive part of the perturbative quark-triangle diagrams, with two heavy quark lines corresponding to the weak $Q \to Q'$ vertex and one light quark line connecting the heavy meson interpolating current vertices in Eq. (54). At the next-to-leading order in $\alpha_s$, all possible internal gluon lines in such triangle diagrams must be considered, as displayed in Fig. 1.
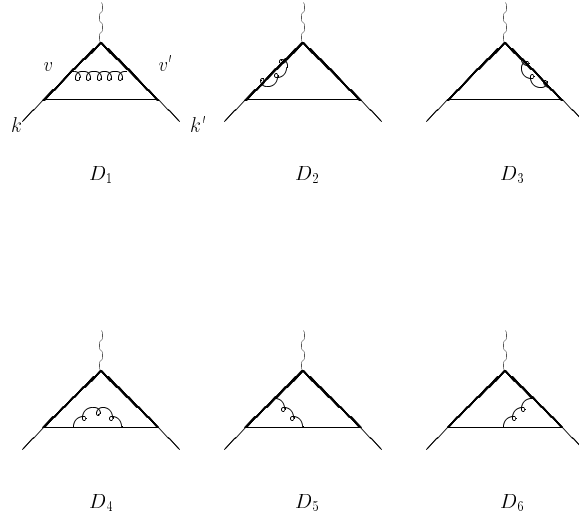


Figure 1: Two-loop diagrams relevant for the calculation of $\mathcal{O}(\alpha_s)$ corrections to the perturbative part of the QCD sum rule for the universal form factors. The heavy lines represent the heavy quark propagators in HQET.

Therefore, for the dispersive representation in Eq. (55) in terms of hadronic intermediate states one assumes the ansatz

$$\Pi(\omega, \omega', y) = \Pi_{pole}(\omega, \omega', y) + \Pi_{continuum}(\omega, \omega', y) \qquad (57)$$

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where, for simplicity, the dependence of the continuum contribution on the thresholds $\nu_c$ and $\nu'_c$ has been omitted.

The correlator $\Pi(\omega, \omega', y)$ can be expressed in QCD in the Euclidean region, i.e. for large negative values of $\omega$ and $\omega'$, in terms of perturbative and nonperturbative contributions

$$\Pi(\omega, \omega', y) = \int d\nu d\nu' \frac{\rho^{pert}(\nu, \nu', y)}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \Pi^{np}(\omega, \omega', y) \quad . \qquad (58)$$

In Eq. (58), $\Pi^{np}$ represents the series of power corrections in the "small" $1/\omega$ and $1/\omega'$ variables, determined by quark and gluon vacuum condensates ordered by increasing dimension. These universal QCD parameters account for general properties of the nonperturbative strong interactions, for which asymptotic freedom cannot be applied. The lowest dimensional ones can be obtained from independent theoretical sources, or fitted from other applications of QCD sum rules in the cases where the hadronic dispersive contribution is particularly well-known.[9] In practice, since the higher dimensional condensates are not known, one truncates the power series and *a posteriori* verifies the validity of such an approximation.

The QCD sum rule is finally obtained by imposing that the two representations of $\Pi(\omega, \omega', y)$, namely the QCD representation (Eq. (58)) and the pole-plus-continuum ansatz (Eq. (57)), match in a suitable range of Euclidean values of $\omega$ and $\omega'$.

A double Borel transform in the variables $\omega$ and $\omega'$

$$\frac{1}{\tau}\hat{\mathcal{B}}_\tau^{(\omega)} = \lim \frac{\omega^n}{(n-1)!}\left(-\frac{d}{d\omega}\right)^n, \qquad (n \to \infty, \omega \to -\infty, \tau = -\frac{\omega}{n} \text{ fixed}) \quad (59)$$

(and similar for $\hat{\mathcal{B}}_{\tau'}$) is applied to "optimize" the sum rule. As a matter of fact, this operation has two effects. The first one consists of factorially improving the convergence of the nonperturbative series, justifying the truncation procedure; the second effect enhances the role of the lowest-lying meson states while minimizing that of the model for the hadron continuum. The *a priori* undetermined mass parameters $\tau$ and $\tau'$ must be chosen in a suitable range of values, expected to be of the order of the typical hadronic mass scale ($\geq 1~GeV$), where the optimization is verified and, in addition, the prediction turns out to be reasonably stable. After the Borel transformation, possible subtraction terms are eliminated and Eq. (55) can be rewritten as

$$\hat{\Pi}(\tau, \tau', y) = \int d\nu d\nu'\, e^{-\left(\frac{\nu}{\tau} + \frac{\nu'}{\tau'}\right)} \rho(\nu, \nu', y) \,. \qquad (60)$$

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Equation (56) shows that the preliminary evaluation of the constants $F_1(\mu)$ and $F_2(\mu)$ is necessary to exploit the sum rule for the determination of $\xi$. This is done within the same HQET QCD sum rule framework, specifically by evaluating two-point functions.

In the following we shall give the results for these quantities focusing on the transitions $B \to (D, D^*)$ and $B \to (D_0, D_1')$.

## 6.1   The Isgur-Wise Function

The main properties of the Isgur-Wise function have been already described in the previous sections. Here we recall the derivation of such form factor using QCD sum rules.[40,33,41] As stressed above, a preliminary part of the analysis consists of the effective theory calculation of the leptonic constant of the heavy mesons belonging to the fundamental doublet $(P, P^*)$, $P = D$, $B$, since it enters in the sum rule for the $\xi$ function through the relation in Eq. (56). We have already met such a parameter in Eq. (39); the constant $F$ in that equation is defined by

$$\langle 0|J_5^v|P(v)\rangle = F(\mu) \ , \tag{61}$$

where $J_5^v = \bar{q}i\gamma_5 h_v$ interpolates the heavy pseudoscalar meson. One considers therefore the two-point correlator

$$\psi(\omega) = i \int d^4x\, e^{ik\cdot x} \langle 0|T\{J_5^{v\dagger}(x)J_5^v(0)\}|0\rangle \ . \tag{62}$$

One could have chosen equally to work with the vector current and the $P^*$ meson, due to the degeneracy of the two members of the doublet.

At the next-to-leading order in renormalization group improved perturbation theory it is possible to define a renormalized scheme-independent quantity as follows:

$$F_{ren} = [\alpha_s(\mu)]^{2/9}\left\{1 - \frac{\alpha_s(\mu)}{\pi}[Z+\delta]\right\} F(\mu) \ , \tag{63}$$

where, in the $\overline{MS}$ scheme,

$$Z = 3\frac{153 - 19n_f}{(33 - 2n_f)^2} - \frac{381 - 30n_f + 28\pi^2}{36(33 - 2n_f)}$$

and $\delta = 2/3$.

The sum rule for $F(\mu)$ allows to evaluate also the parameter $\bar{\Lambda}$. This can be done considering the logarithmic derivative of the sum rule itself with respect to the Borel parameter $\tau'$. The results are [33,46]

$$F_{ren} = 0.40 \pm 0.06 \text{ GeV}^{3/2} , \qquad \bar{\Lambda} = 0.57 \pm 0.07 \text{ GeV} . \tag{64}$$

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The important result is that radiative corrections modify by $\sim 30\%$ the value of $F_{ren}$, a result which is not specific of QCD sum rules, but reflects a general property of the considered two-point correlator, and whose origin could be traced back to a Coulomb interaction between quarks, since the largest enhancement comes from the contribution of gluon exchange between the heavy and the light quark. The important role of radiative corrections in the numerical evaluation of $F$ is confirmed by other analyses.[47,48]

The QCD sum rule result for $F$ can be used as an input in the three-point function in Eq. (54) in order to compute the Isgur-Wise function. In this case one can choose the two Dirac structures $\Gamma_1$, $\Gamma_2$ to interpolate the ground state heavy mesons $(P, P^*)$. Therefore, the choices $\Gamma_{1,2} = i\gamma_5$ (corresponding to the pseudoscalar meson $P$) or $\Gamma_{1,2} = \gamma_\mu - v_\mu$ (corresponding to the vector meson $P^*$) are equally acceptable, since the structure resulting from the computation of the trace will factor out in all the terms contributing to the sum rule. Finally, in Eq. (54), we choose $J_W = h_{v'}^{Q'} \gamma_\mu (1 - \gamma_5) h_v^Q$, in order to describe the weak process.

In the lowest order in the perturbation theory, the sum rule fulfills the important constraint of reproducing the zero-recoil point normalization of the Isgur-Wise function,[40,33] a result which continues to hold after the inclusion of the $\alpha_s$ correction.[41] Hence, we observe that the QCD sum rules formulated in the framework of the effective theory automatically incorporate the correct normalization of the function $\xi$. This observation stresses the suitability of employing this technique in such a context. Furthermore, the inclusion of radiative corrections to a three-point correlator of heavy quark currents has been done for the first time in the HQET, where the task is simplified due to the simpler Feynman rules. The calculation of the loop integrals required to evaluate the diagrams in Fig. 1 can be done using integration by parts, in such a way as to reduce complicated integrals to simpler ones by recursive relations, and the method of differential equations developed by Kotikov.[49] The inclusion of radiative QCD corrections to the quark condensate had been done before the full inclusion of such corrections in the perturbative term.[33] Also it has been shown [41] that the calculation of the relevant loop integrals is easier when dealing directly with the Borel transformed sum rule. In the case of the Isgur-Wise function, the symmetry of the correlator allows to set equal the two Borel parameters: $\tau = \tau'$, a simplification which is not allowed in the case of the transitions of the $B$ meson to excited states, as we shall see in the next subsection.

Let us observe that the currents employed in Eq. (54) are those defined in the effective theory and therefore are subject to renormalization. When computing any universal form factor, in Eq. (54) there appear two heavy-light

currents and a heavy-heavy current; in the $\overline{MS}$ scheme the renormalization constants of such currents are

$$\mathcal{Z}_{hl} = 1 - \frac{\alpha_s}{2\pi\hat{\epsilon}}\,, \qquad \mathcal{Z}_{hh} = 1 + \frac{\alpha_s}{2\pi\hat{\epsilon}}\gamma(y) \tag{65}$$

where

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln 4\pi, \tag{66}$$

the parameter $\epsilon$ being defined in the $D$-dimensional space-time by

$$D = 4 + 2\epsilon,$$

reflecting the use of dimensional regularization in the employed renormalization scheme. $\gamma(y)$ has been defined in the previous section. The inclusion of $\alpha_s$ corrections to the three-point correlator should reproduce the singularity structure of these renormalization constants. As a matter of fact, this is explicitly verified both in the case of the $\xi$ function and for the function $\tau_{1/2}$ to be considered in the next section.

We will not report here the sum rule for $\xi$, referring the reader to the original literature.[41] What is worth noticing is that the radiative corrections turn out to be small, due to their cancelation in the ratio of three- to two-point functions. For this reason not only the normalization of $\xi$ is preserved, but also the impact of QCD corrections is modest for large recoil.

In Fig. 2 we show the QCD sum rule outcome[41] for the renormalized scheme-independent Isgur-Wise function.[b] It is possible to fit the curve by writing

$$\xi(y) = \xi(1) \left[1 - \rho^2 \, (y - 1)\right] \,\,, \tag{67}$$

where $\rho^2$ is known as the slope of the $\xi$ function. An analogous decomposition holds for the function $\xi_{ren}$. Hence, the sum rule predicts [12]

$$\rho^2_{ren} = 0.66 \pm 0.05 \,. \tag{68}$$

This result is in agreement with the others available in literature.[51,52] In particular, as shown by Neubert,[50] it lies within the bounds which can be obtained on the basis of two sum rules, the Bjorken sum rule[53] and the Voloshin sum rule.[54,55] Such two sum rules will be discussed in the next section.
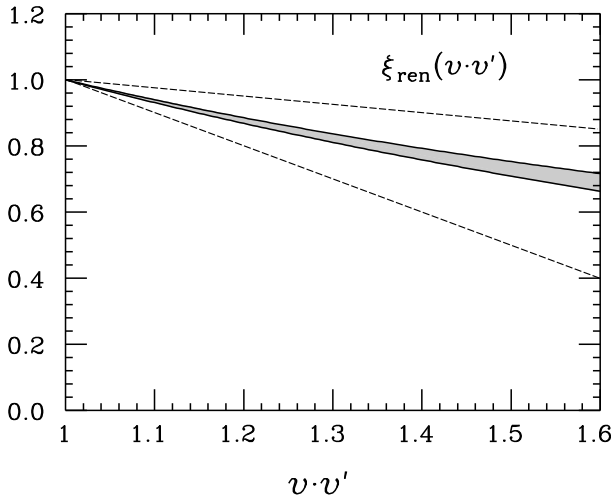
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[b]The figure was taken from a review by M. Neubert.[50]

Figure 2: QCD sum rule result for the Isgur-Wise function in the range of $y = v \cdot v'$ allowed in $B \to D^{(*)}$ transitions. The dashed lines indicate the bounds on the slope at $y = 1$ derived applying the Bjorken and Voloshin sum rules.

## 6.2 B decays to Excited Charmed Resonances

It is worth analyzing other cases analogous to the determination of the Isgur-Wise form factor $\xi$, in particular it is interesting to calculate the universal form factors governing the semileptonic $B$ meson decays into the positive parity charmed excited states. These higher-lying states correspond to the $L = 1$ orbital excitations in the non-relativistic constituent quark model. Besides their theoretical relevance to HQET,[56] in particular to the aspects of the QCD sum rule calculations, such $B \to D^{**}$ semileptonic transitions (from now on $D^{**}$ will indicate the generic $L = 1$ charmed state) have numerous additional points of physical interest. Indeed, in principle these decay modes may account for a sizeable fraction of semileptonic $B$-decays, and consequently they represent a well-defined set of corrections to the theoretical prediction that the total semileptonic $B \to X_c$ decay rate should be saturated by the $B \to D$ and $B \to D^*$ modes,[6] in the limit $m_Q \to \infty$ and under the condition $(m_b - m_c)/(m_b + m_c) \to 0$ (the so-called small-velocity limit). Moreover, the shape of the inclusive differential decay distribution in the lepton energy could reflect contributions from the $B \to D^{**}$ modes.

The issue of the contribution of such modes to the $B$ semileptonic branching ratio $\mathcal{B}_{SL}$ is also relevant since theoretical determinations of this quantity

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stay above the experimental data. In fact, the parton model gives $\mathcal{B}_{SL} \simeq 15\%$, with negligible $1/m_Q$ corrections.$^c$ Perturbative QCD corrections have a more relevant impact, nevertheless leaving $\mathcal{B}_{SL} \geq 12.5\%$. Also the experimental results seemed controversial for a long time, since the data obtained at the $\Upsilon(4S)$ by the CLEO Collaboration were different from those obtained by the LEP Collaborations at the $Z^0$ peak. Recent results[57] show an improved situation,

$$
\begin{array}{rcll}
\mathcal{B}(B \to X\ell\nu_\ell) & = & (10.63 \pm 0.17)\% & (Z^0 \text{ average}) , \\
\mathcal{B}(B \to X\ell\nu_\ell) & = & (10.45 \pm 0.21)\% & (\Upsilon(4S) \text{ average}) ;
\end{array} \tag{69}
$$

the last number is the Particle Data Group average for the data at the $\Upsilon(4S)$.[17] A possible reduction of the theoretical prediction could be obtained increasing the value of $n_c$, i.e. the number of charmed hadrons produced in the $B$ decay. However, since the various experimental data for these parameter show a very good agreement both among themselves and with theoretical predictions, it is likely that the solution should be found elsewhere. In particular, it is important to fully understand which is the contribution of the various possible decay modes, for example, those with $D^{**}$ in the final state.

Another important result, relevant both to phenomenology and to the critical tests of HQET, is the relation of the $B \to D^{**}$ form factors at zero recoil to the slope $\rho^2$ of the $B \to D^{(*)}$ Isgur-Wise function, through the Bjorken sum rule,[53]

$$
\rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}|^2 + 2 \sum_m |\tau_{3/2}^{(m)}|^2 . \tag{70}
$$

In this equation $n$, $m$ identify the radial excitations of states with the same $J^P$. The $D^{**}$ states considered here represent the lowest lying states contributing to the left-hand side of Eq. (70).

Of similar interest for HQET is the test of the upper bound on such universal form factors at zero recoil, involving the heavy meson "binding energy" and the $D^{**} - D$ mass splittings,[54,55]

$$
\bar{\Lambda} = 2 \left( \sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2 + 2 \sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 \right) , \tag{71}
$$

where $\epsilon_k = M_{H_Q^{(k)}} - M_{P_Q}$.

Besides, the investigation of the semileptonic $B$ transitions to excited charm states is an important preliminary study for the theoretical analysis

---

$^c$As we shall see in the next section, the parton model result identifies with the leading term in the $1/m_Q$ expansion for inclusive decay rates.

of the production of such states in nonleptonic $B$ decays,[58,18] as well as for the identification of additional decay modes (such as $D^{(*)}D^{(*)}\pi$) suitable for the investigation of CP violating effects at $B$ factories.[59]

Finally, as a byproduct of the QCD sum rule calculation, theoretical predictions for the $D^{**}$ meson masses can be obtained, which are obviously interesting *per se*.

We have already introduced two universal form factors $\tau_{1/2}$, $\tau_{3/2}$ describing the $B$ meson transitions to the members of the two doublets with spin-parity $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$, $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ respectively. Let us consider now the HQET QCD sum rule calculation of the function $\tau_{1/2}$, which has been performed at the next-to-leading order in the renormalization group improved perturbation theory.[43] The analogous result for the function $\tau_{3/2}$ is available only at tree level in perturbative QCD corrections.[42,44,60]

The matrix elements of the semileptonic $B \to D_0 \ell \bar{\nu}$ and $B \to D'_1 \ell \bar{\nu}$ transitions can be parameterized in terms of six form factors,

$$
\begin{aligned}
\frac{\langle D_0(v')|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle}{\sqrt{m_B m_{D_0}}} &= g_+(v+v')_\mu + g_-(v-v')_\mu \,, \\
\frac{\langle D'_1(v',\epsilon)|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle}{\sqrt{m_B m_{D'_1}}} &= g_{V_1}\epsilon^*_\mu + \epsilon^* \cdot v \left[g_{V_2}v_\mu + g_{V_3}v'_\mu\right] , \\
&\quad - i\, g_A \epsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v^\beta v'^\gamma \quad,
\end{aligned}
\tag{72}
$$

where $v$ and $v'$ are four-velocities and $\epsilon$ is the $D'_1$ polarization vector. The form factors $g_i$ depend on the variable $y = v \cdot v'$, which is directly related to the momentum transfer to the lepton pair. Again, the heavy quark spin symmetry allows us to relate the form factors $g_i(y)$ in Eq. (72) to a single function $\tau_{1/2}(y)$[56] through short-distance coefficients, perturbatively calculable, which depend on the heavy quark masses $m_b, m_c$, on $y$ and on a mass-scale $\mu$. The heavy quark spin symmetry allows us to connect the QCD vector and axial-vector currents to the HQET currents. At the next-to-leading logarithmic approximation in $\alpha_s$ and in the infinite heavy quark mass limit, the relations between $g_i$ and $\tau_{1/2}$ are given by

$$
\begin{aligned}
g_+(y) + g_-(y) &= -2\left(C^5_1(y,\mu) + (y-1)C^5_2(y,\mu)\right)\tau_{1/2}(y,\mu) \,, \\
g_+(y) - g_-(y) &= 2\left(C^5_1(y,\mu) - (y-1)C^5_3(y,\mu)\right)\tau_{1/2}(y,\mu) \,, \\
g_{V_1}(y) &= 2(y-1)\,C_1(y,\mu)\,\tau_{1/2}(y,\mu) \,, \\
g_{V_2}(y) &= -2\,C_2(y,\mu)\,\tau_{1/2}(y,\mu) \,, \\
g_{V_3}(y) &= -2\left(C_1(y,\mu) + C_3(y,\mu)\right)\tau_{1/2}(y,\mu) \,,
\end{aligned}
$$

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$$g_A(y) \quad = \quad -2\,C_1^5(y,\mu)\,\tau_{1/2}(y,\mu)\;. \tag{73}$$

Following the same steps leading to Eq. (47), a renormalization-group invariant form factor can be defined

$$\tau_{1/2}^{ren}(y) = K_{hh}(y,\mu)\tau_{1/2}(y,\mu)\;; \tag{74}$$

hence, in Eq. (73) one can substitute $C_i^{(5)}(y,\mu)$ by $\hat{C}_i^{(5)}(y)$ and $\tau_{1/2}(y,\mu)$ by $\tau_{1/2}^{ren}(y)$. Analogous relations hold for the eight form factors parameterizing the matrix elements of $B \to D_1\ell\bar{\nu}$ and $B \to D_2\ell\bar{\nu}$; in this case the heavy quark symmetry allows to relate them to the universal function $\tau_{3/2}(y)$.[56] The main difference with respect to the Isgur-Wise form factor $\xi(y)$ is that one cannot invoke symmetry arguments to predict the normalization of both $\tau_{1/2}(y)$ and $\tau_{3/2}(y)$, and therefore a calculation of the form factors in the whole kinematical range is required. For $B \to (D_0, D_1')\ell\bar{\nu}$ the physical range for the variable $y$ is restricted between $y = 1$ and $y = 1.309 - 1.326$, taking into account the values for the mass of $D_0$, $D_1'$ ($m_{D_0, D_1'} = 2.40 - 2.45$ GeV).

Let us consider the transition $B \to D_0$ and define

$$\langle 0|J_s^v|P_0(v)\rangle = F^+(\mu)\;\;, \tag{75}$$

where $J_s^v = \bar{q}h_{v'}^{Q'}$ represents the local interpolating current of the scalar $(D_0)$ meson. In analogy to Eq. (16), we can write, in the heavy quark limit,

$$\bar{\Lambda}^+ = M_{D_0} - m_c\;\;. \tag{76}$$

By considering the following two-point correlator:

$$\Psi(\omega') = i \int d^4x\; e^{ik'\cdot x}\langle 0|T[J_s^{v'}(x)J_s^{v'}(0)^\dagger]|0\rangle\;\;, \tag{77}$$

it is possible to obtain a sum rule for the constant $F^+(\mu)$. A $\mu$-independent constant $F_{ren}^+$ can be defined, using the relation between $F^+(\mu)$ and the matrix element of the scalar current in full QCD,

$$F_{ren}^+ = [\alpha_s(\mu)]^{\frac{d}{2}}\left[1 - \frac{\alpha_s(\mu)}{\pi}\;Z\right]F^+(\mu) \tag{78}$$

where $Z$ has been defined in the previous subsection.

Again, the sum rule for $F^+(\mu)$ allows to evaluate the parameter $\bar{\Lambda}^+$, by considering the logarithmic derivative with respect to the Borel parameter $\tau'$. The following predictions are obtained:[43]

$$\bar{\Lambda}^+ = 1.0 \pm 0.1 \text{ GeV}\,, \qquad F_{ren}^+ = 0.7 \pm 0.2 \text{ GeV}^{\frac{3}{2}}\;. \tag{79}$$

Perturbative $\mathcal{O}(\alpha_s)$ corrections represent a sizeable contribution to the sum rule for $F^+_{ren}$, similar to the situation met in the case of $F$.[47,33,48]

Considering again the three point correlator Eq. (54) and using

$$J_1^{v'} = J_s^{v'} = \bar{q}h_{v'}^c, \quad J_W = \tilde{A}_\mu = \bar{h}_{v'}^c \gamma_\mu \gamma_5 h_v^b, \quad \text{and} \quad J_2^v = J_5^v = \bar{q}i\gamma_5 h_v^b,$$

it is possible to derive the sum rule for the form factor $\tau_{1/2}^{ren}$. Also in this case the $\alpha_s$ correction in the perturbative term is sizeable, but it turns out to be partially compensated by the analogous correction in the leptonic constants $F$, $F^+$. Notice that this is a remarkable result, not expected *a priori*, since the normalization of the form factor, for example at zero recoil, is not fixed by symmetry arguments. The perturbative corrections, however, do not equally affect the form factor for all values of the variable $y$, but they are sensibly $y$ dependent, with the effect of increasing the slope of $\tau_{1/2}$ with respect to the case where they are omitted. The result is depicted in Fig. 3,[43] where the curves refer to various choices for the continuum thresholds. The region limited by the curves essentially determines the theoretical accuracy of the calculation. Considering the $y$ dependence, the limited range of values allowed by the mass difference between $D$ and $D_0$ permits the expansion near $y = 1$,

$$\tau_{1/2}^{ren}(y) = \tau_{1/2}(1)\left(1 - \rho_{1/2}^2(y-1) + c_{1/2}(y-1)^2\right) \quad . \tag{80}$$

A three-parameter fit to Fig. 3 gives

$$\tau_{1/2}(1) = 0.35 \pm 0.08 \; , \qquad \rho_{1/2}^2 = 2.5 \pm 1.0 \; , \qquad c_{1/2} = 3 \pm 3 \; . \tag{81}$$

An immediate application of this result concerns the prediction of the semileptonic $B$ decay rates to $D_0$ and $D_1'$. Using $V_{cb} = 3.9 \times 10^{-2}$ and $\tau(B) = 1.56 \times 10^{-12}$ sec, one obtains

$$\mathcal{B}(B \to D_0 \ell\bar{\nu}) = (5 \pm 3) \times 10^{-4}, \qquad \mathcal{B}(B \to D_1' \ell\bar{\nu}) = (7 \pm 5) \times 10^{-4} \quad . \tag{82}$$

This means that only a very small fraction of the semileptonic $B \to X_c$ decays is represented by transitions into the $s_\ell^P = \frac{1}{2}^+$ charmed doublet.

The $B$ meson transitions to final charmed states belonging to the $s_\ell = 3/2$ doublet are described by the universal form factor $\tau_{3/2}(y)$. This is defined by the following matrix elements in the heavy quark limit:

$$\langle D_1(v', \epsilon)|\bar{h}_{v'}^c \gamma_\mu (1-\gamma_5) h_v^b|B(v)\rangle = \sqrt{m_B m_{D_1}} \; \tau_{3/2}(v \cdot v')$$

$$\times \left\{ \left[ \frac{(1-(v \cdot v')^2)}{\sqrt{2}} \; \epsilon_\mu^* + \frac{(\epsilon^* \cdot v)}{\sqrt{2}} [-3 \; v_\mu + (v \cdot v' - 2) \; v_\mu'] \right. \right.$$

$$\left. \left. +i\frac{(v \cdot v\prime - 1)}{2\sqrt{2}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu}(v+v')^\alpha \cdot (v-v')^\beta \right] \right\} , \tag{83}$$
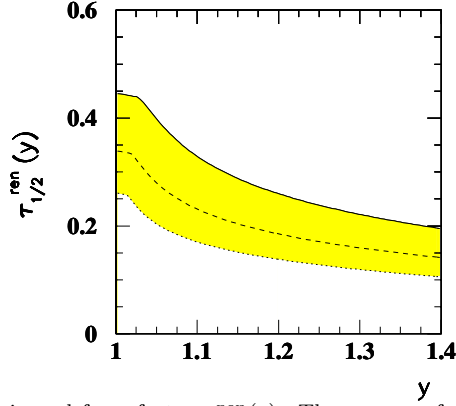
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Figure 3: The universal form factor $\tau_{1/2}^{ren}(y)$. The curves refer to choices of the threshold parameters: $\nu_c = 2.0$ GeV, $\nu'_c = 2.5$ GeV (continuous line), $\nu_c = 2.5$ GeV, $\nu'_c = 3.0$ GeV (dashed line), $\nu_c = 3.0$ GeV, $\nu'_c = 3.5$ GeV (dotted line).

$$\langle D_2(v',\epsilon)|\bar{h}^c_{v'}\gamma_\mu(1-\gamma_5)h^b_v|B(v)\rangle = \sqrt{m_B m_{D_2}}\ \tau_{3/2}(v\cdot v')$$

$$\times [\ i\ \frac{\sqrt{3}}{2}\epsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha\nu}v_\nu(v+v')^\beta(v-v')^\gamma$$

$$-\sqrt{3}(v\cdot v'+1)\epsilon^*_{\mu\alpha}v^\alpha + \sqrt{3}\epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\mu]. \tag{84}$$

The QCD sum rule analysis for this function is available in the leading order in $\alpha_s$.[42] The procedure closely follows the one outlined above, with the interpolating currents

$$J_v^{\mu\nu} = \frac{1}{\sqrt{6}}\bar{q}(\overleftrightarrow{\partial^\mu}\gamma^\nu + \overleftrightarrow{\partial^\nu}\gamma^\mu - \frac{1}{2}g^{\mu\nu}\overleftrightarrow{\partial^\rho}\gamma_\rho)h_v$$

chosen to interpolate the $D_2$ meson with $J^P = 2^+$, and

$$\tilde{J}_v^\mu = \bar{q}\gamma_5\overleftrightarrow{\partial^\mu}h_v$$

interpolating the $D_1$ meson with $J^P = 1^+$,

$$\langle 0|J_v^{\mu\nu}|D_2(p,\epsilon)\rangle = \epsilon_{\mu\nu}F^+_{3/2}\sqrt{m_c}\ ,$$

$$\langle 0|\tilde{J}_v^\mu|D_1(p,\epsilon)\rangle = \epsilon_\mu F^+_{3/2}\sqrt{m_c}\ . \tag{85}$$

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The mass parameter analogous to those defined in Eqs. (16), (76) is given by

$$M_{D_2,D_1} = m_c + \bar{\Lambda}_{3/2} \quad . \tag{86}$$

The sum rule analysis for these quantities gives[42]

$$\bar{\Lambda}_{3/2} = 1.05 \pm 0.10 \text{ GeV} , \qquad F_{3/2}^+ = 0.43 \pm 0.06 \text{ GeV}^{5/2} . \tag{87}$$

As for the form factor $\tau_{3/2}(y)$, the result of the sum rule is shown in Fig. 4; it corresponds to

$$\tau_{3/2}(y) = \tau_{3/2}(1)[1 - \rho_{3/2}^2(y - 1)] \tag{88}$$

with $\tau_{3/2}(1) \simeq 0.4$ and $\rho_{3/2}^2 \simeq 0.52$. These values lead to the predictions

$$\mathcal{B}(B \to D_1 \ell \bar{\nu}) \simeq 3.2 \times 10^{-3} , \qquad \mathcal{B}(B \to D_2 \ell \bar{\nu}) \simeq 4.8 \times 10^{-3} \quad . \tag{89}$$
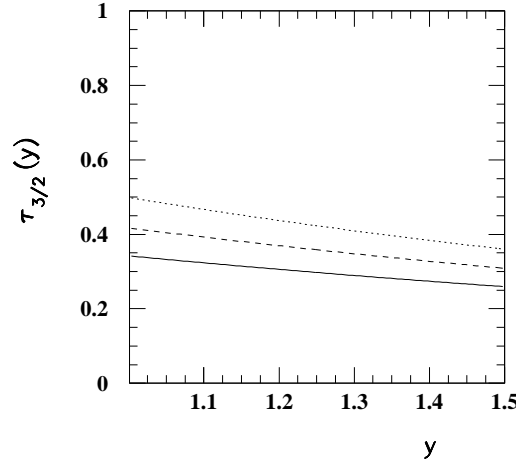


Figure 4: The universal form factor $\tau_{3/2}(y)$ The three curves refer to different choices of the threshold parameters in the perturbative contribution to the sum rule.

Other determinations of the $\tau$ functions have appeared in the literature, employing various versions of the constituent quark model[61] The results range in quite a large interval, $\tau_{1/2}(1) = 0.06 - 0.40$, $\rho_{1/2}^2 = 0.7 - 1.0$ and $\tau_{3/2}(1) =$

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$0.31 - 0.66$, $\rho_{3/2}^2 = 1.4 - 2.8$. These results critically depend on the features of the models employed.

Finally, let us mention that the universal form factors describing the $B$ meson transitions to the higher doublets with $s_\ell^P = \frac{3}{2}^-$ and $s_\ell^P = \frac{5}{2}^-$ have also been computed by QCD sum rules.[62] Such doublets comprise the states with $J_{s_\ell}^P = (1^-, 2^-)_{3/2}$ and $J_{s_\ell}^P = (2^-, 3^-)_{3/2}$, respectively. In this case, the application of the method to high-spin states shows several difficulties, mainly due to the identification of the range of the parameters needed in the sum rule analyses, because of the particular features of the considered states and of their interpolating currents. Some information could be obtained from other theoretical approaches, namely constituent quark models predicting the heavy meson spectrum. The final result, although affected by a sizeable theoretical uncertainty, nevertheless is useful for assessing the role of high-spin meson doublets in constituting a part of the charm inclusive semileptonic $B$ decay width. Denoting the members of the considered doublets by $(D_1^*, D_2^*)$, $(D_2^{*\prime}, D_3^*)$ corresponding to $s_\ell^P = 3/2^-$ and $s_\ell^P = 5/2^-$, respectively, one finds that [62]

$$\mathcal{B}(B \to D_2^{*\prime} \ell \nu_\ell) \simeq \mathcal{B}(B \to D_3^* \ell \nu_\ell) \simeq 1 \times 10^{-5} \ . \qquad (90)$$

On the other hand, the $B$ decays to the $s_\ell^P = 3/2^-$ doublet turn out to be negligible at the leading order in the $1/m_Q$ expansion, due to the small value of the corresponding universal form factor.

At the end of our discussion about the universal form factors describing $B$ decays in the $m_Q \to \infty$ limit, it should be stressed that an important issue concerns the role of $1/m_Q$ corrections, which introduce subleading form factors. The analysis of such corrections has been carried out for a number of relevant cases, and we refer the reader to the relevant literature.[12,63,64]

## 7   Inclusive Heavy Hadron Decays

We have outlined above how the use of heavy quark symmetries allows a simplified description of exclusive heavy hadron decays, both introducing universal form factors and allowing to classify the symmetry-breaking contributions using an expansion in the inverse heavy quark mass.[55] The presence of a large mass also turns out to be very useful when dealing with inclusive decays, i.e. the transitions of a given particle into all possible final states with assigned quantum numbers. Such decays represent theoretically "clean" processes, since it is expected that in the sum over many hadrons the bound state effects, which are the main source of uncertainty in the calculation of exclusive transitions, should be eliminated by the averaging procedure. This expectation is based on the quark-hadron duality.[45] Besides, the possibility of computing inclusive

decay rates of heavy hadrons by means of an expansion in the inverse powers of the heavy quark mass provides in this case a reliable QCD-based systematic approach.

Let us briefly summarize the main aspects of such QCD analysis of the inclusive decay widths of the heavy hadrons. The starting point is the transition operator $\hat{T}(Q \to X_f \to Q)$,[65]

$$\hat{T} = i \int d^4x \; T[\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)] \tag{91}$$

describing an amplitude with the heavy quark $Q$ having the same momentum in the initial and final state. $\mathcal{L}_W$ is the effective weak Lagrangian governing the decay $Q \to X_f$. The inclusive width of the hadron $H_Q$ can be obtained by averaging $\hat{T}$ over $H_Q$ and taking the imaginary part of the forward matrix element,

$$\Gamma(H_Q \to X_f) = \frac{2 \; Im \; \langle H_Q | \hat{T} | H_Q \rangle}{2 \; M_{H_Q}} \; . \tag{92}$$

The reason behind calculating the right-hand side of Eq. (92) is to set up an operator product expansion for the transition operator $\hat{T}$ in terms of local operators $\mathcal{O}_i$,

$$\hat{T} = \sum_i C_i \mathcal{O}_i \tag{93}$$

with $\mathcal{O}_i$ ordered according to their dimension, and the coefficients $C_i$ containing appropriate inverse powers of the heavy quark mass $m_Q$. The lowest dimension operator appearing in Eq. (93) is $\mathcal{O}_3 = \bar{Q}Q$. The next gauge and Lorentz invariant operator is the $D = 5$ chromomagnetic operator $\mathcal{O}_G = \bar{Q}\frac{g}{2}\sigma_{\mu\nu}G^{\mu\nu}Q$.

The matrix element of $\bar{Q}Q$ over $H_Q$ can be obtained using the heavy quark equation of motion, expanded in the heavy quark mass,

$$\bar{Q}Q = \bar{Q}\gamma^0 Q + \frac{\mathcal{O}_G}{2m_Q^2} - \frac{\mathcal{O}_\pi}{2m_Q^2} + O(m_Q^{-3}), \tag{94}$$

where $\mathcal{O}_\pi$ is the kinetic energy operator $\mathcal{O}_\pi = \bar{Q}(i\vec{D})^2 Q$. On the other hand, the $H_Q$ matrix element of $\bar{Q}\gamma^0 Q$ is unity (modulo the covariant normalization of the states).

The number of independent operators appearing in Eq. (93) increases if the $1/m_Q^3$ term is considered. Such operators are of the four-quark type

$$\mathcal{O}_6^q = \bar{Q}\Gamma q \; \bar{q}\Gamma Q \tag{95}$$

where $\Gamma$ is an appropriate combination of the Dirac and color matrices.

In this way, a complete classification of various contributions to the inclusive decay rates can be obtained for different hadrons $H_Q$. In the expression for the inclusive width $\Gamma(H_Q \to X_f)$

$$\Gamma(H_Q \to X_f) = \Gamma_0^f \left[ A_0^f + \frac{A_2^f}{m_Q^2} + \frac{A_3^f}{m_Q^3} + ... \right] \qquad (96)$$

the $A_i^f$ factors which (together with $\Gamma_0^f$) depend on the final state $X_f$, include perturbative short-distance coefficients and nonperturbative hadronic matrix elements incorporating the long range dynamics. The partonic prediction for the width in Eq. (96) corresponds to the leading term $\Gamma^{part}(H_Q \to X_f) = \Gamma_0^f A_0^f$, with $A_0^f = 1 + c^f \alpha_s/\pi + O(\alpha_s^2)$ and $\Gamma_0^f \propto m_Q^5$; differences among the widths of the hadrons $H_Q$ emerge at the next to leading order in $1/m_Q$ and are related to different values of the matrix elements of the operators $\mathcal{O}_i$ of dimension larger than three.

It is important to notice the absence of the first order term $m_Q^{-1}$ in Eq. (96), a result obtained by Chay, Georgi and Grinstein,[66] and Bigi, Uraltsev and Vainshtein.[67] This depends on the adopted choice of the definition of the heavy quark mass, as explained in Sec. 3.

The occurrence of operators of the type (95) is an appealing feature of the expansion in Eq. (93), as far as the determination of the inclusive widths is concerned. As a matter of fact, contrary to the $D = 5$ operators $\mathcal{O}_G$ and $\mathcal{O}_\pi$ which are spectator blind, the $D = 6$ operators give different contributions when averaged over hadrons belonging to the same $SU(3)$ light flavor multiplet, and therefore they are responsible of different lifetimes of, e.g., $B^-$ and $B_s$, $\Lambda_b$ and $\Xi_b$. The spectator flavor dependence is related to the mechanisms of weak scattering and Pauli interference,[65] both suppressed by the factor $m_Q^{-3}$ with respect to the parton decay rate.

In the following subsection, we shall see how the $1/m_Q$ expansion provides a systematic framework to discuss one of the open questions in the $b$ phenomenology, i.e. the problem of the ratios of beauty hadron lifetimes.

### 7.1   The Problem of Beauty Hadron Lifetime Ratios

An interesting problem of present-day heavy quark physics is represented by the measured difference between the $\Lambda_b$ baryon and $B_d$ meson lifetimes, $\tau(\Lambda_b^0) = 1.208 \pm 0.051 \; ps$ and $\tau(\bar{B}^0) = 1.548 \pm 0.021 \; ps$.[68] As a matter of fact, the deviation from unity, at the level of 20%, of the ratio $\tau(\Lambda_b)/\tau(B_d)$, $\tau(\Lambda_b^0)/\tau(\bar{B}^0) = 0.780 \pm 0.037$,[d] is in contradiction with the naive expectation

---

[d]These data were reported by the Particle Data Group on the basis of the analyses reported by the LEP $B$ Lifetime Group.[68]

that, at the scale of the $b$ quark mass, the spectator model should describe rather accurately the decays of the hadrons containing one heavy quark. The ratio $\tau(\Lambda_b)/\tau(B_d)$ can be computed in QCD using the previously described approach. We have already noticed that the first term of the expansion in Eq. (96) reproduces the parton model result, and therefore it contributes universally to the inclusive decay width of all hadrons containing the same heavy quark. Besides, we also pointed out that the flavor of the spectator quark is first felt at $\mathcal{O}(m_Q^{-3})$.

As for the differences in the lifetime of mesons and baryons, they could already arise at the level $m_Q^{-2}$, both due to the chromomagnetic contribution and to the kinetic energy term in Eq. (92). In particular, the kinetic energy term is responsible for the difference for systems where the chromomagnetic contribution vanishes, namely in the case of $\Lambda_b$ and $\Xi_b$ (the light degrees of freedom in $S$ wave). However, the results of a calculation of $\mu_\pi^2$ for mesons [69] and baryons [70] support the conjecture [71] that the kinetic energy operator has practically the same matrix element when computed on such hadronic systems. The approximate equality of the kinetic energy operator on $B_d$ and $\Lambda_b$ can also be inferred by considering that, to the leading order in $1/m_Q$, $\mu_\pi^2(\Lambda_b)$ can be related [72] to $\mu_\pi^2(B_d)$ and to the heavy quark masses by an expression which assumes the charm mass $m_c$ heavy enough for a meaningful expansion in $1/m_c$. Namely,

$$
\begin{aligned}
\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d) &\simeq \frac{m_b m_c}{2(m_b - m_c)} \\
&\times \left[ (M_B + 3M_{B^*} - 4M_{\Lambda_b}) - (M_D + 3M_{D^*} - 4M_{\Lambda_c}) \right].
\end{aligned} \tag{97}
$$

Using present data and the CDF measurement $M_{\Lambda_b} = 5623 \pm 5 \pm 4$ MeV [73] Eq. (97) gives $\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d) \simeq 0.002 \pm 0.024$ GeV$^2$, where the error mainly comes from the error in $M_{\Lambda_b}$. The QCD sum rule outcome for $\mu_\pi^2(H_Q)$ is $\mu_\pi^2(B_d) \simeq \mu_\pi^2(\Lambda_b) \simeq 0.6$ GeV$^2$, with an estimated uncertainty of about 30%.[69,70] This result implies that the differences between the meson and baryon lifetimes should occur at the $m_Q^{-3}$ level, thus involving the four-quark operators in Eq. (95). They can be classified as follows:[74]

$$
\begin{aligned}
O_{V-A}^q &= \bar{Q}_L \gamma_\mu q_L \, \bar{q}_L \gamma_\mu Q_L \,, \\
O_{S-P}^q &= \bar{Q}_R q_L \, \bar{q}_L Q_R \,, \\
T_{V-A}^q &= \bar{Q}_L \gamma_\mu \frac{\lambda^a}{2} q_L \, \bar{q}_L \gamma_\mu \frac{\lambda^a}{2} Q_L \,, \\
T_{S-P}^q &= \bar{Q}_R \frac{\lambda^a}{2} q_L \, \bar{q}_L \frac{\lambda^a}{2} Q_R \,,
\end{aligned} \tag{98}
$$

39

with $q_{R,L} = \dfrac{1 \pm \gamma_5}{2} q$ and $\lambda_a$ the Gell-Mann matrices.

For mesons, the vacuum saturation approximation can be used to compute the matrix elements of the operators in Eq. (98),

$$
\begin{aligned}
\langle B_q | O^q_{V-A} | B_q \rangle_{VSA} &= \Big( \frac{m_b + m_q}{M_{B_q}} \Big)^2 \langle B_q | O^q_{S-P} | B_q \rangle_{VSA} = \frac{f^2_{B_q} M^2_{B_q}}{4} \ , \\
\langle B_q | T^q_{V-A} | B_q \rangle_{VSA} &= \langle B_q | T^q_{S-P} | B_q \rangle_{VSA} = 0 \ \ .
\end{aligned}
\tag{99}
$$

Therefore, the matrix elements are expressed in terms of quantities such as $f_B$ and the quark masses, and the resulting numerical values can be used in the calculation of the lifetimes, with the only caveat concerning the accuracy of the factorization approximation.[74]

The vacuum saturation approach cannot be employed for baryons; in this case a direct calculation of the matrix elements is required, for example using constituent quark models.

A simplification can be obtained [74] for $\Lambda_b$ using color and Fierz identities and introducing the operators

$$
\tilde{\mathcal{O}}^q_{V-A} = \bar{Q}^i_L \gamma_\mu Q^i_L \ \bar{q}^j_L \gamma^\mu q^j_L
\tag{100}
$$

and

$$
\tilde{\mathcal{O}}^q_{S-P} = \bar{Q}^i_L q^j_R \ \bar{q}^j_L Q^i_R
\tag{101}
$$

($i$ and $j$ are color indices). As a matter of fact, the $\Lambda_b$ matrix elements of the operators in Eq. (98) can be expressed in terms of $\langle \Lambda_b | \tilde{\mathcal{O}}^q_{V-A} | \Lambda_b \rangle$ and $\langle \Lambda_b | O^q_{V-A} | \Lambda_b \rangle$, modulo $1/m_Q$ corrections contributing to subleading terms in the expression for the inclusive widths.

The matrix element of $\tilde{\mathcal{O}}^q_{V-A}$ and $\mathcal{O}^q_{V-A}$ can be parametrized as

$$
\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_b} = \frac{\langle \Lambda_b | \tilde{\mathcal{O}}^q_{V-A} | \Lambda_b \rangle}{2 M_{\Lambda_b}} = \frac{f^2_B M_B}{48} r
\tag{102}
$$

and

$$
\langle \Lambda_b | O^q_{V-A} | \Lambda_b \rangle = -\tilde{B} \ \langle \Lambda_b | \tilde{\mathcal{O}}^q_{V-A} | \Lambda_b \rangle
\tag{103}
$$

with $\tilde{B} = 1$ in the valence quark approximation.

For $f_B = 200$ MeV and $r = 1$, Eq. (102) corresponds to the value: $\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_b} = 4.4 \times 10^{-3}$ GeV$^3$. The $\Lambda_c$ matrix element of $\tilde{\mathcal{O}}^q_{V-A}$ has been computed using a bag model and a nonrelativistic quark model;[75] the results $\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_c} \simeq 0.75 \times 10^{-3}$ GeV$^3$ and $\langle \tilde{\mathcal{O}}^q_{V-A} \rangle_{\Lambda_c} \simeq 2.5 \times 10^{-3}$ GeV$^3$, correspond to $r \simeq 0.2$ and $r \simeq 0.6$, respectively. A different model [76] is used in another analysis.[77]

Larger values of the matrix elements have been advocated by Rosner[78] using the values of the mass splitting $\Sigma_b^* - \Sigma_b$ and $\Sigma_c^* - \Sigma_c$, and assuming that the $\Lambda_b$ and $\Sigma_b$ wave functions are similar, $r \simeq 0.9 \pm 0.1$, taking $M_{\Sigma_b^*}^2 - M_{\Sigma_b}^2 = M_{\Sigma_c^*}^2 - M_{\Sigma_c}^2$, or $r \simeq 1.8 \pm 0.5$ using the DELPHI measurement.[79]

Information from constituent quark models have been supplemented by estimates based on field theoretical approaches, such as QCD sum rules and lattice. As a matter of fact, a large value of $r$, namely $r \simeq 4 - 5$, would explain the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$.[74] The application of the QCD sum rule method to the calculation of the matrix element of an operator of high dimension presents a number of disadvantages; nevertheless, interesting and quite reliable information can be obtained. The result [80] is $\langle \Lambda_b | \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b \rangle \simeq (0.4 - 1.20) \times 10^{-3}$ GeV$^3$, corresponding to the parameter $r$ in the range $r \simeq 0.1 - 0.3$.[e] The conclusion of such analysis is that the inclusion of $1/m_Q^3$ terms in the expression of the inclusive widths does not solve the puzzle represented by the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. As a matter of fact, using the formulae for the lifetime ratio in the framework of the heavy quark expansion,[74] the QCD sum rule result together with $\tilde{B} = 1$ gives

$$\tau(\Lambda_b)/\tau(B_d) \geq 0.94 \quad . \tag{104}$$

As for lattice QCD, a calculation with static $b$ quark finds higher values for $r$, $r \simeq 1.2 \pm 0.2$, which nevertheless is not enough to explain the observed discrepancy in the lifetime ratio.[82]

In conclusion, the expansion in the inverse heavy quark mass provides a systematic framework to study inclusive heavy hadron decays. In particular it allows to compute the corrections to the partonic picture, which identifies with the first term of such an expansion, in terms of the matrix elements of suitable operators, weighted by increasing powers of $m_Q^{-1}$. However, the application of such a formalism to the computation of the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ does not explain yet the observed ratio, leaving this issue as one of the exciting open problems in the phenomenology of beauty hadrons.

## 8 Conclusions and Perspectives

Weak decays of heavy quarks are especially appealing, mainly because the presence of a large scale, i.e. $m_Q$, allows to formulate model independent relations among nonperturbative quantities which are difficult to access in full

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[e]Another QCD sum rule analysis [81] finds a larger value for the parameter $r$. However, this result strongly depends on the assumption of a huge deviation from the vacuum saturation approximation for the four-quark condensate.

QCD. We have shown what we consider among the most relevant phenomenological examples: the use of heavy quark symmetry in parametrizing weak matrix elements between heavy hadrons in terms of universal form factors, and the application of $1/m_Q$ expansion to compute reliably inclusive decay rates of heavy hadrons. In both cases, the most straightforward application is the extraction of the CKM matrix elements $V_{cb}$ and $V_{ub}$. The former has been determined with increasing accuracy, the latter, being intrinsically smaller, is still a subject of ongoing analyses.

Many interesting topics have not been touched upon. The list includes rare $B$ decays, among which those induced by flavor changing neutral current represent a fertile ground for exploring physics beyond the Standard Model; these are commonly viewed as those processes which can tell us something about new physics before the LHC era. A remarkable example are radiative decays induced by the $b \rightarrow s$ transition, measured both in the inclusive and exclusive $B \rightarrow K^* \gamma$ modes,[17,83] which put significant constraint on the space of parameters of various new physics scenarios.

Another untouched sector concerns nonleptonic decays, where the impact of nonperturbative QCD effects is the highest. The effective Hamiltonian we have given in Sec. 2 is the starting point for the theoretical treatment of these processes. It allows us to sort out the nonperturbative quantities, the hadronic matrix elements of the operators in the OPE, from the perturbative ones, i.e. the Wilson coefficients. Having identified them, the problem of giving a reliable estimate of such quantities is still unsolved. Usually, either models or simplified approaches have been employed, among which a widely used one is factorization, either in its simpler formulation, *naive* factorization,[84] or in its most refined versions.[85] This is quite a general problem for all nonleptonic decays, which is not specific of heavy hadrons. Nevertheless, the situation seems once more easier in this case, since it turns out that factorization works correctly in the heavy quark limit, at least for those decays usually classified as *Class I*.[86] This is probably (and hopefully) a field in growth.

Nonleptonic decays play a central role in the analyses of CP violation in $B$ decays, since the most promising channels to extract the angles of the unitarity triangle are just nonleptonic processes. The issue of CP violation is one of the most extensively reviewed, and we refer the reader to the existing literature.[87]

Finally, I have not discussed the $B_s$ and $B_c$ decays, which will play a role at the CERN Large Hadron Collider (LHC).[88]

It seems that in the very last years the ongoing experiments in high-energy physics are providing us with many exciting new results, from the SuperKamiokande observation of the atmospheric neutrino anomaly,[89] to the new measurements [90] of $\mathrm{Re}(\epsilon'/\epsilon)$, to the very recent direct observation of the

$\nu_\tau$ in July 2000.[91]

As for $B$ physics, the $B$-factories have provided us with the very first results in summer 2000, thus opening the season of intense investigations in this sector of the elementary interactions. In a few years, the advent of LHC will tell us even more. The heavy hadron physics presents a number of exciting perspectives as well as intriguing problems still to be solved, such as the one of the beauty hadron lifetime ratios. Moreover, the precise measurements of CP violating processes and of the phases of the CKM matrix elements will provide us with many answers and, at the same time, will open new problems.

In 1990, J.D. Bjorken[53] pointed out that there was a great potential in the discovery of new symmetries in the heavy quark sector. I hope to have shown that he was right.

## Acknowledgments

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